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Chapter 21

Rolling Element Bearings

Symbols

$\tilde{A}$	constant	$\tilde{G}$	speed effect parameter
B	total conformity ratio	$\tilde{H}_m$	misalignment factor
b	contact semiwidth, m	i	number of rows of rolling elements
$\tilde{C}$	load rating of bearing, N	$\tilde{J}$	number of stress cycles
c	radial clearance, m	K_1	constant defined in Eq. (21.34)
c_d	diametral clearance, m	$K_{1.5}$	constant defined in Eq. (21.33)
c_r	distance between race centers, m	k	ellipticity parameter, D_y/D_x
$\tilde{D}_x$	diameter of contact ellipse along x -axis, m	L_A	life adjustment
$\tilde{D}_y$	diameter of contact ellipse along y -axis, m	$\tilde{L}$	fatigue life
$\tilde{D}$	material factor	$\tilde{L}_{10}$	fatigue life for probability of survival of 0.90
d	diameter of rolling element, m	ℓ	length of rolling element, m
d_b	bore diameter of rolling-element bearing, m	ℓ_ℓ	roller effective length, m
d_e	pitch diameter, m	ℓ_t	roller length, m
d_i, d_o	inner- and outer-race diameter, m	ℓ_v	length dimension in stressed volume, m
E	modulus of elasticity, Pa	$\tilde{M}$	probability of failure
E'	effective elastic modulus, $2 \left(\frac{1 - \nu_a^2}{E_a} + \frac{1 - \nu_b^2}{E_b} \right)^{-1}$, Pa	m_k	load-life exponent
$\tilde{E}$	metallurgical processing factor	N_c	cycle of failures
$\mathcal{E}$	complete elliptic integral of second kind	n	number of rolling elements per row
e_1	constant	P	dimensionless pressure
$\tilde{F}_\ell$	lubrication factor	p	pressure, Pa
$\mathcal{F}$	complete elliptic integral of first kind	R	curvature sum, m
f_c	coefficient depending on material and bearing type	R_r	race conformity, $r/2r_r$
		R_x, R_y	effective radii in x and y directions, respectively, m
		r	radius, m
		r_c	corner radius, m

r_r	crown radius, m		height, deg
$\tilde{S}$	probability of survival	λ_f	dimensionless friction force
s_h	shoulder height, m	λ_h	dimensionless elastic energy
u	velocity in x direction, m/s		lost from hysteresis
u_a	surface a velocity, m/s	μ	coefficient of sliding friction
u_b	surface b velocity, m/s	μ_r	coefficient of rolling friction
$\tilde{V}$	elemental volume, m ³	τ	shear stress, Pa
v_i, v_o	linear velocities of inner and outer contacts, m/s	τ_u	endurance strength, Pa
w_e	bearing equivalent load, N	ψ	angle to load line, deg
w_t	total thrust load of bearing, N	ψ_ℓ	angular extent of bearing load, deg
w_x	tangential load, N	ω	angular velocity, rad/s
w_z	normal load, N	ω_b	angular velocity of rolling element about its axis, rad/s
$\tilde{X}, \tilde{Y}$	factors for calculating equivalent load	ω_c	angular velocity of separator or ball set, rad/s
Z_w	constant, see Eq. (21.46)	ω_i, ω_o	angular velocity of inner and outer races, rad/s
z_0	depth of maximum shear stress, m	ω_r	angular velocity of race, rad/s
z'	stress-weighted mean depth, m		
β	contact angle, deg		
β_f	free contact angle, deg		
γ_d	angle defined in Fig. 21.9, deg		
δ_m	elastic deformation where $dp/dx = 0$, m		
$\delta_{\max}$	total elastic compression on load line, m		
δ_ψ	elastic compression of ball, m		
θ_s	angle used to define shoulder		

Subscripts

a	solid a
b	solid b
i	inner
o	outer
x, y, z	Cartesian coordinates

21.1 Introduction

Since Chap. 16, the focus has been on nonconformal lubricated conjunctions and the development of elastohydrodynamic lubrication theory. In this and the following chapter the principles developed in those chapters are applied to specific machine elements. This chapter focuses on rolling-element bearings, which are a precise, yet simple machine element of great utility, and draws together the current understanding of these bearings. First, the history of rolling-element bearings is briefly reviewed. Subsequent sections are devoted to describing the types of rolling-element bearing, their geometry and kinematics, the materials they are made from, and the manufacturing processes involved. The organization of this chapter is such that unloaded and unlubricated rolling-element bearings are considered in the first five sections, loaded but unlubricated rolling-element bearings in Secs. 21.7 and 21.8 (as well as Chapter 17), and loaded and lubricated rolling-element bearings in Secs. 21.9 to 21.12 (as well as Chapters 18 to 20).

21.2 Historical Overview

The purpose of a bearing is to provide relative positioning and rotational freedom while transmitting a load between two structures, usually a shaft and a housing. The basic form and concept of the rolling-element bearing are simple. If loads are to be transmitted between surfaces in relative motion in a machine, the action can be facilitated in a most effective manner if rolling elements are interposed between the sliding members. The frictional resistance encountered in sliding is then largely replaced by the much smaller resistance associated with rolling, although the arrangement is inevitably afflicted with high stresses in the restricted regions of effective load transmission.

The precision rolling-element bearing of the twentieth century is a product of exacting technology and sophisticated science. It is simple in form and concept and yet extremely effective in reducing friction and wear in a wide range of machinery. The spectacular development of numerous forms of rolling-element bearing in the twentieth century is well known and documented, but it is possible to trace the origins and development of these vital machine elements to periods long before there was a large industrial demand for such devices and certainly long before adequate machine tools for their effective manufacture existed in large quantities. A complete historical development of rolling-element bearings is given in Hamrock and Dowson (1981), and therefore only a brief overview is presented here.

The influence of general technological progress on the development of rolling-element bearings, particularly those concerned with the movement of heavy stone building blocks and carvings, road vehicles, precision instruments, water-raising equipment, and windmills is discussed in Hamrock and Dowson (1981). The concept of rolling-element bearings emerged in embryo form in Roman times, faded from the scene during the Middle Ages, was revived during the Renaissance, developed steadily in the seventeenth and eighteenth centuries for various applications, and was firmly established for individual road carriage bearings during the Industrial Revolution. Toward the end of the nineteenth century, the great merit of ball bearings for bicycles promoted interest in the manufacture of accurate steel balls. Initially, the balls were turned from bars on special lathes, with individual general machine manufacturing companies making their own bearings. Growing demand for both ball and roller bearings encouraged the formation of specialist bearing manufacturing companies at the turn of the century and thus laid the foundations of a great industry. The advent of precision grinding techniques and the availability of improved materials did much to confirm the future of the new industry.

The essential features of most forms of modern rolling-element bearing were therefore established by the second half of the nineteenth century, but it was the formation of specialist precision-manufacturing companies in the early years of the twentieth century that finally established the rolling-element bearing as a most valuable, high-quality, readily available machine component. The availability of ball and roller bearings in standard sizes has had a tremendous impact

on machine design throughout the twentieth century. Such bearings still provide a challenging field for further research and development, and many engineers and scientists are currently engaged in exciting and demanding research projects in this area. In many cases, new improved materials or enlightened design concepts have extended the life and range of application of rolling-element bearings, yet in other respects, much remains to be done in explaining the extraordinary operating characteristics of bearings that have served our technology so well for almost a century. A recent development, covered in Chapters 18 to 20, is the understanding and analysis of one important aspect of rolling-element performance—the lubrication mechanism in small, highly stressed conjunctions between the rolling element and the rings, or races.

21.3 Bearing Types

Ball and roller bearings are available to the designer in a great variety of designs and size ranges. The intent of this chapter is not to duplicate the complete descriptions given in manufacturers' catalogs but rather to present a guide to representative bearing types along with the approximate range of sizes available. Tables 21.1 to 21.9 illustrate some of the more widely used bearing types. In addition, numerous types of specialty bearing are available; space does not permit a complete cataloging of all available bearings. Size ranges are generally given in metric units. Traditionally, most rolling-element bearings have been manufactured to metric dimensions, predating the efforts toward a metric standard.

In addition to bearing types and approximate size ranges available, Tables 21.1 to 21.9 also list approximate relative load-carrying capacities, both radial and thrust, and, where relevant, approximate tolerances to misalignment.

Rolling-element bearings are an assembly of several parts: an inner race, an outer race, a set of balls or rollers, and a cage or separator. The cage or separator maintains even spacing of the rolling elements. A cageless bearing, in which the annulus is packed with the maximum rolling-element complement, is called a "full-complement bearing." Full-complement bearings have high load carrying capacity but lower speed limits than bearings equipped with cages. Tapered-roller bearings are an assembly of a cup, a cone, a set of tapered rollers, and a cage.

21.3.1 Ball Bearings

Ball bearings are used in greater quantity than any other type of rolling-element bearing. For an application where the load is primarily radial with some thrust load present, one of the types in Table 21.1 can be chosen. A Conrad, or deep-groove, bearing has a ball complement limited by the number of balls that can be packed into the annulus between the inner and outer races with the inner race resting against the inside diameter of the outer race. A stamped and riveted two-piece cage, piloted on the ball set, or a machined two-piece

Table 21.1: Characteristics of representative radial ball bearings [*From Hamrock and Anderson (1983)*].

Type		Approximate range of bore sizes, mm		Relative capacity		Limiting speed factor	Tolerance to misalignment
		Minimum	Maximum	Radial	Thrust		
Conrad or deep groove		3	1060	1.00	^a 0.7	1.0	±0°15'
Maximum capacity or filling notch		10	130	1.2-1.4	^a 0.2	1.0	±0°3'
Magneto or counterbored outer		3	200	0.9-1.3	^b 0.5-0.9	1.0	±0°5'
Airframe or aircraft control		4.826	31.75	High static capacity	^a 0.5	0.2	0°
Self-aligning, internal		5	120	0.7	^b 0.2	1.0	±2°30'
Self-aligning, external		—	—	1.0	^a 0.7	1.0	High
Double row, maximum		6	110	1.5	^a 0.2	1.0	±0°3'
Double row, deep groove		6	110	1.5	^a 1.4	1.0	0°

^a Two directions.

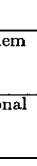
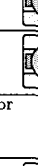
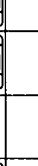
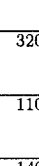
^b One direction.

cage, ball-piloted or race-piloted, is almost always used in a Conrad bearing. The only exception is a one-piece cage with open-sided pockets that is snapped into place. A filling-notch bearing most often has both inner and outer races notched so that a ball complement limited only by the annular space between the races can be used. It has low thrust capacity because of the filling notch.

The self-aligning internal bearing shown in Table 21.1 has an outer-race ball path ground in a spherical shape so that it can accept high levels of misalignment. The self-aligning external bearing has a multipiece outer race with a spherical interface. It too can accept high misalignment and has higher capacity than the self-aligning internal bearing. However, the self-aligning external bearing is somewhat less self-aligning than its internal counterpart because of friction in the multipiece outer race.

Representative angular-contact ball bearings are illustrated in Table 21.2. An angular-contact ball bearing has a two-shouldered ball groove in one race and a single-shouldered ball groove in the other race. Thus, it is capable of supporting only a unidirectional thrust load. The cutaway shoulder allows

Table 21.2: Characteristics of representative angular-contact ball bearings (Minimum bore size, 10 mm) *From Hamrock and Anderson (1983)*].

Type		Approximate maximum bore size, mm	Relative capacity		Limiting speed factor	Tolerance to misalignment
			Radial	Thrust		
One-directional thrust		320	^b 1.00-1.15	^{a, b} 1.5-2.3	^b 1.1-3.0	±0°2'
Duplex, back to back		320	1.85	^c 1.5	3.0	0°
Duplex, face to face		320	1.85	^c 1.5	3.0	0°
Duplex, tandem		320	1.85	^a 2.4	3.0	0°
Two directional or split ring		110	1.15	^c 1.5	3.0	±0°2'
Double row		140	1.5	^c 1.85	0.8	0°
Double row, maximum		110	1.65	^a 0.5 ^d 1.5	0.7 ^d 1.5	0°

^aOne direction.
^bDepends on contact angle.
^cTwo directions.
^dIn other direction.

Table 21.3: Characteristics of representative thrust ball bearings (Relative radial capacity, 0) *From Hamrock and Anderson (1983)*].

Type		Approximate range of bore sizes, mm		Relative thrust capacity	Limiting speed factor	Tolerance to misalignment
		Minimum	Maximum			
One directional, flat race		6.45	88.9	^a 0.7	0.10	^b 0°
One directional, grooved race		6.45	1180	^a 1.5	0.30	0°
Two directional, grooved race		15	220	^c 1.5	0.30	0°

^aOne direction.
^bAccepts eccentricity.
^cTwo directions.

Table 21.4: Characteristics of representative cylindrical roller bearings. [*From Hamrock and Anderson (1983)*].

Type		Approximate range of bore sizes, mm		Relative capacity		Limiting speed factor	Tolerance to misalignment
		Minimum	Maximum	Radial	thrust		
Separable outer ring, nonlocating (RN, RIN)		10	320	1.55	0	1.20	$\pm 0^\circ 5'$
Separable inner ring, nonlocating (RU, RIU)		12	500	1.55	0	1.20	$\pm 0^\circ 5'$
Separable outer ring, one-direction locating (RF, RIF)		40	177.8	1.55	^a Locating	1.15	$\pm 0^\circ 5'$
Separable inner ring, one-direction locating (RJ, RIJ)		12	320	1.55	^a Locating	1.15	$\pm 0^\circ 5'$
Self-contained, two-direction locating		12	100	1.35	^b Locating	1.15	$\pm 0^\circ 5'$
Separable inner ring, two-direction locating (RT)(RIT)		20	320	1.55	^b Locating	1.15	$0^\circ 5'$
Nonlocating, full complement (RK, RIK)		17	75	2.10	0	0.20	$\pm 0^\circ 5'$
Double row, separable outer ring, nonlocating (RD)		30	1060	1.85	0	1.00	0°
Double row, separable inner ring, nonlocating		70	1060	1.85	0	1.00	0°

^aOne direction.

^bTwo directions.

assembly of the bearing by snapping the inner race over the ball set after it is positioned in the cage and outer race. This also permits use of a one-piece, machined, race-piloted cage that can be balanced for high-speed operation. Typical contact angles vary from 15 to 25°.

Angular-contact ball bearings are used in duplex pairs mounted either back to back or face to face as shown in Table 21.2. Duplex bearing pairs are manufactured so that they “preload” each other when clamped together in the housing and on the shaft. The use of preloading provides stiffer shaft support and helps prevent bearing skidding at light loads. Proper levels of preload can be obtained from the manufacturer. A duplex pair can support bidirectional thrust load. The back-to-back arrangement offers more resistance to moment or overturning loads than does the face-to-face arrangement.

Where thrust loads exceed the capability of a simple bearing, two bearings can be used in tandem, with both bearings supporting part of the thrust load.

Table 21.5: Characteristics of representative spherical roller bearings. [From Hamrock and Anderson (1983)].

Type		Approximate range of bore sizes, mm		Relative capacity		Limiting speed factor	Tolerance to mis-alignment
		Mini-mum	Maxi-mum	Radial	Thrust		
Single row, barrel or convex		20	320	2.10	0.20	0.50	±2°
Double row, barrel or convex		25	1250	2.40	0.70	0.50	±1°30'
Thrust		85	360	^a 0.10 ^b 0.10	^a 1.80 ^b 2.40	0.35-0.50	±3°
Double row, con-cave		50	130	2.40	0.70	0.50	±1°30'

^aSymmetric rollers.

^bAsymmetric rollers.

Three or more bearings are occasionally used in tandem, but this is discouraged because of the difficulty in achieving good load sharing. Even slight differences in operating temperature will cause a maldistribution of load sharing.

The split-ring bearing shown in Table 21.2 offers several advantages. The split ring (usually the inner) has its ball groove ground as a circular arc with a shim between the ring halves. The shim is then removed when the bearing is assembled so that the split-ring ball groove has the shape of a gothic arch. This reduces the axial play for a given radial play and results in more accurate axial positioning of the shaft. The bearing can support bidirectional thrust loads but

Table 21.6: Characteristics of standardized double-row, spherical roller bearings. [From Hamrock and Anderson (1983)].

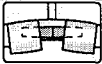
Type	Roller design	Retainer design	Roller guidance	Roller-race contact
SLB 	Symmetric	Machined, roller piloted	Retainer pockets	Modified line, both races
SC 	Symmetric	Stamped, race piloted	Floating guide ring	Modified line, both races
SD 	Asymmetric	Machined, race piloted	Inner-ring center rib	Line contact, outer; point contact, inner

Table 21.7: Characteristics of spherical roller bearings. [*From Hamrock and Anderson (1983).*]

Series	Types	Approximate range of bore sizes, mm		Approximate relative capacity ^a		Limiting speed factor
		Minimum	Maximum	Radial	Thrust	
202	Single-row barrel	20	320	1.0	0.11	0.5
203	Single-row barrel	20	240	1.7	0.18	0.5
204	Single-row barrel	25	110	2.1	0.22	0.4
212	SLB	35	75	1.0	0.26	0.6
213	SLB	30	70	1.7	0.53	0.6
22, 22K	SLB, SC, SD	30	320	1.7	0.46	0.6
23, 23K	SLB, SC, SD	40	280	2.7	1.0	0.6
30, 30K	SLB, SC, SD	120	1250	1.2	0.29	0.7
31, 31K	SLB, SC, SD	110	1250	1.7	0.54	0.6
32, 32K	SLB, SC, SD	100	850	2.1	0.78	0.6
39, 39K	SD	120	1250	0.7	0.18	0.7
40, 40K	SD	180	250	1.5	—	0.7

^a Load capacities are comparative within the various series of spherical roller bearings only. For a given envelope size, a spherical roller bearing has a radial capacity approximately equal to that of a cylindrical roller bearing.

must not be operated for prolonged periods of time at predominantly radial loads. This restriction results in three-point ball-race contact and relatively high frictional losses. As with the conventional angular-contact bearing, a one-piece precision-machined cage is used.

Ball thrust bearings (90° contact angle), Table 21.3, are used almost exclusively for machinery with vertically oriented shafts. The flat-race bearing allows eccentricity of the fixed and rotating members. An additional bearing must be used for radial positioning. It has low load-carrying capacity because of its small ball-race contact and consequent high Hertzian stress. Grooved-race bearings have higher load-carrying capacities and are capable of supporting low-magnitude radial loads. All the pure thrust ball bearings have modest speed capability because of the 90° contact angle and the consequent high level of ball spinning and frictional losses.

21.3.2 Roller Bearings

Cylindrical roller bearings, Table 21.4, provide purely radial load support in most applications. An N or U type of bearing will allow free axial movement of the shaft relative to the housing to accommodate differences in thermal growth. An F or J type of bearing will support a light thrust load in one direction; and a T type of bearing a light bidirectional thrust load. Cylindrical roller bearings have moderately high radial load-carrying capacity as well as high-speed capability. Their speed capability exceeds that of both spherical and tapered-roller bearings. A commonly used bearing combination for support of a high-speed

rotor is an angular-contact ball bearing or duplex pair and a cylindrical roller bearing. As explained in the following section on bearing geometry, the rollers in cylindrical roller bearings are seldom pure cylinders. They are crowned, or made slightly barrel shaped, to relieve stress concentrations on the roller ends when any misalignment of the shaft and housing is present. Cylindrical roller bearings may be equipped with one- or two-piece cages, usually race piloted. For greater load-carrying capacity, full complement bearings can be used, but at a significant sacrifice in speed capability.

Spherical roller bearings, Tables 21.5 to 21.7, are made as either single- or double-row bearings. The more popular bearing design uses barrel-shaped rollers. An alternative design employs hourglass-shaped rollers. Spherical roller bearings combine extremely high radial load-carrying capacity with modest thrust load-carrying capacity (with the exception of the thrust type) and excellent tolerance to misalignment. They find widespread use in heavy-duty rolling mill and industrial gear drives, where all these bearings characteristics are requisite.

Tapered-roller bearings, Table 21.8, are also made as single- or double-row bearings with combinations of one- or two-piece cups and cones. A four-row bearing assembly with two- or three-piece cups and cones is also available. Bearings are made with either a standard angle for applications in which moderate thrust loads are present or with a steep angle for high thrust capacity. Standard and special cages are available to suit the application requirements. Single-row tapered-roller bearings must be used in pairs because a radially loaded bearing generates a thrust reaction that must be taken up by a second bearing. Tapered-roller bearings are normally set up with spacers designed so that they operate with some internal play. Manufacturers' engineering journals should be consulted for proper setup procedures.

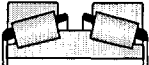
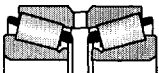
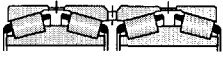
Needle roller bearings, Table 21.9, are characterized by compactness in the radial direction and are frequently used without an inner race. In the latter case the shaft is hardened and ground to serve as the inner race. Drawn cups, both open and closed end, are frequently used for grease retention. Drawn cups are thin walled and require substantial support from the housing. Heavy-duty roller bearings have relatively rigid races and are more akin to cylindrical roller bearings with long-width-to-diameter-ratio rollers.

Needle roller bearings are more speed limited than cylindrical roller bearings because of roller skewing at high speeds. A high percentage of needle roller bearings are full-complement bearings. They have higher load-carrying capacity than a caged needle bearing, but lower speed capability.

Many types of specialty bearing are available other than those discussed here. Aircraft bearings for control systems, thin-section bearings, and fractured-ring bearings are some of the more widely used bearings among the many types manufactured. Complete coverage of all bearing types is beyond the scope of this chapter.

Angular-contact ball bearings and cylindrical roller bearings are generally considered to have the highest speed capabilities. Speed limits of roller bearings

Table 21.8: Characteristics of representative tapered-roller bearings. [*From Hamrock and Anderson (1983)*].

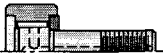
Type		Subtype	Approximate range of bore sizes, mm	
			Minimum	Maximum
Single row (TS)		TS, straight bore TST, tapered bore TSS, steep angle TSE, TSK, keyway cone TSF, TSSF, flanged cup	8 24 16 12 8	1690 430 1270 380 1070
Two row, double cone, single cup (TDI)		TDI, straight bore TDIK, TDIT, TDITP, tapered bore TDIE, TDIKE, slotted double cone TDIS, steep angle	30 30 24 55	1200 860 690 520
Two row, double cup, single cone, adjustable (TDO)		TDO TDO, steep angle	8 20	1830 1430
Two row, double cup, single cone, nonadjustable (TNA)		TNA, straight bore TNASW, slotted cone TNASWE, extended cone rib TNASWH, slotted cone, sealed	20 30 20 8	60 260 305 70
Four row, cup adjusted (TQO)		TQO, straight bore TQOT, tapered bore	70 250	1500 1500

are discussed in conjunction with lubrication methods. The lubrication system employed has as great an influence on bearing limiting speed as the bearing design.

21.4 Geometry

The operating characteristics of a rolling-element bearing depend greatly on its diametral clearance. This clearance varies for the different types of bearing discussed in the preceding section. In this section the principal geometrical relationships governing the operation of unloaded rolling-element bearings are developed. This information will be of vital interest when such quantities as stress, deflection, load-carrying capacity, and life are considered in subsequent sections. Although bearings rarely operate in the unloaded state, an understanding of this section is essential to appreciation of the remaining sections.

Table 21.9: Characteristics of representative needle roller bearings. [From Hamrock and Anderson (1983)].

Type	Bore size, mm		Relative load capacity		Limiting speed factor	Misalign- ment tolerance
	Mini- mum	Maxi- mum	Dynamic	Static		
Drawn cup, needle  Open end Closed end	3	185	High	Moderate	0.3	Low
Drawn cup, needle grease retained 	4	25	High	Moderate	0.3	Low
Drawn cup, roller  Open end Closed end	5	70	Moderate	Moderate	0.9	Moderate
Heavy-duty roller 	16	235	Very high	Moderate	1.0	Moderate
Caged roller 	12	100	Very high	High	1.0	Moderate
Cam follower 	12	150	Moderate to high	Moderate to high	0.3-0.9	Low
Needle thrust 	6	105	Very high	Very high	0.7	Low

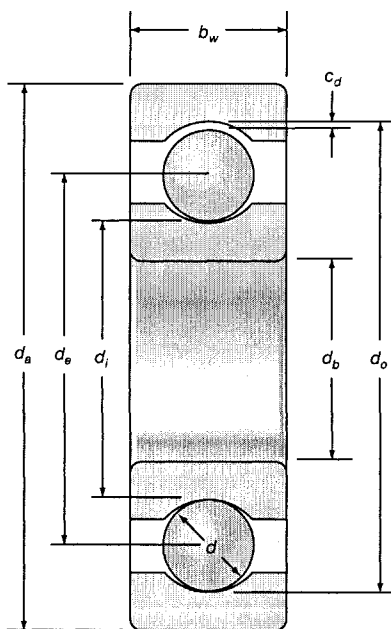


Figure 21.1: Cross section through radial single-row ball bearings. [From Hamrock and Anderson (1983)].

21.4.1 Geometry of Ball Bearings

The geometry of ball bearings involves the elements of pitch diameter and clearance, race conformity, contact angle, endplay, shoulder height, and curvature sum and difference.

Pitch Diameter and Clearance

The cross section through a radial single-row ball bearing shown in Fig. 21.1 depicts the radial clearance and various diameters. The pitch diameter d_e is the mean of the inner- and outer-race contact diameters and is given by

$$d_e = d_i + \frac{1}{2} (d_o - d_i)$$

or

$$d_e = \frac{1}{2} (d_o + d_i) \quad (21.1)$$

Also from Fig. 21.1 the diametral clearance, denoted by c_d , can be written as

$$c_d = d_o - d_i - 2d \quad (21.2)$$

Diametral clearance may therefore be thought of as the maximum distance that one race can move diametrically with respect to the other when no measurable force is applied and both races lie in the same plane. Although diametral

clearance is generally used in connection with single-row radial bearings, Eq. (21.2) is also applicable to angular-contact bearings.

Race Conformity

Race conformity is a measure of the geometrical conformity of the race and the ball in a plane passing through the bearing axis, which is a line passing through the center of the bearing perpendicular to its plane and transverse to the race. Figure 21.2 is a cross section of a ball bearing showing race conformity, expressed as

$$R_r = \frac{r}{d} \quad (21.3)$$

For perfect conformity, where the race radius is equal to the ball radius, R_r is equal to 1/2. The closer the race conforms to the ball, the greater the frictional heat within the contact. On the other hand, open-race curvature and reduced geometrical conformity, which reduce friction, also increase the maximum contact stresses and consequently reduce the bearing fatigue life. For this reason most ball bearings made today have race conformity ratios in the range $0.51 \leq R_r \leq 0.54$, with $R_r = 0.52$ being the most common value. The race conformity ratio for the outer race is usually made slightly larger than that for the inner race to compensate for the closer conformity in the plane of the bearing between the outer race and the ball than between the inner race and the ball. This tends to equalize the contact stresses at the inner- and outer-race contacts. The difference in race conformities does not normally exceed 0.02.

Contact Angle

Radial bearings have some axial play, since they are generally designed to have a diametral clearance, as shown in Fig. 21.3. This implies a free contact angle different from zero. Angular-contact bearings are specifically designed to operate under thrust loads. The clearance built into the unloaded bearing, along with the race conformity ratio, determines the bearing free contact angle. Figure 21.3 shows a radial bearing with contact due to the axial shift of the inner and outer races when no measurable force is applied.

Before the free contact angle is discussed, it is important to define the distance between the centers of curvature of the two races in line with the

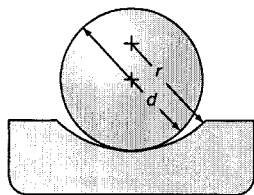


Figure 21.2: Cross section of ball and outer race, showing race conformity. [From Hamrock and Anderson (1983)].

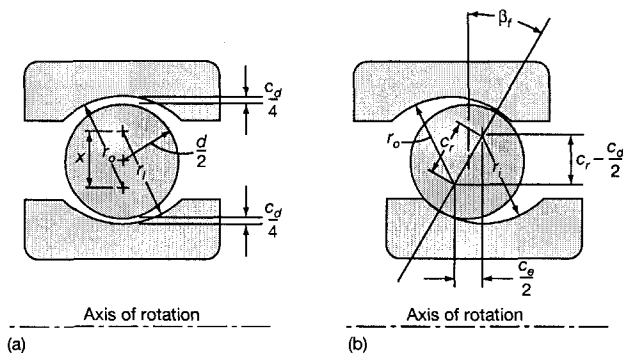


Figure 21.3: Cross section of radial bearing, showing ball-race contact due to axial shift of inner and outer races. (a) Initial position; (b) shifted position. [From Hamrock and Anderson (1983)].

center of the ball in Fig. 21.3a and b. This distance—denoted by x in Fig. 21.3a and by c_r in Fig. 21.3b—depends on race radius and ball diameter. When quantities referred to the inner and outer races are denoted by subscripts i and o , respectively, Fig. 21.3a and b shows that

$$\frac{c_d}{4} + d + \frac{c_d}{4} = r_o - x + r_i$$

or

$$x = r_o + r_i - d - \frac{c_d}{2}$$

and

$$d = r_o - c_r + r_i$$

or

$$c_r = r_o + r_i - d \quad (21.4)$$

From these equations

$$x = c_r - \frac{c_d}{2}$$

This distance, shown in Fig. 21.3a, will be useful in defining the contact angle.

Equation (21.3) can be used to write Eq. (21.4) as

$$c_r = Bd \quad (21.5)$$

where

$$B = R_{r,o} + R_{r,i} - 1 \quad (21.6)$$

The quantity B in Eq. (21.6) is known as the “total conformity ratio” and is a measure of the combined conformity of both the outer and inner races to the

ball. Calculations of bearing deflection in later sections depend on the quantity B .

The free contact angle β_f (Fig. 21.3b) is defined as the angle made by a line through the points where the ball contacts both races and a plane perpendicular to the bearing axis of rotation when no measurable force is applied. Note that the centers of curvature of both the outer and inner races lie on the line defining the free contact angle. From Fig. 21.3b the expression for the free contact angle can be written as

$$\cos \beta_f = 1 - \frac{c_d}{2c_r} \quad (21.7)$$

Equations (21.2) and (21.4) can be used to write Eq. (21.7) as

$$\beta_f = \cos^{-1} \frac{r_o + r_i - \frac{1}{2}(d_o - d_i)}{r_o + r_i - d} \quad (21.8)$$

Equation (21.8) shows that if the size of the balls is increased and everything else remains constant, the free contact angle is decreased. Similarly, if the ball size is decreased, the free contact angle is increased.

From Eq. (21.7) the diametral clearance c_d can be written as

$$c_d = 2c_r (1 - \cos \beta_f) \quad (21.9)$$

This is an alternative definition of the diametral clearance given in Eq. (21.2).

Endplay

Free endplay c_e is the maximum axial movement of the inner race with respect to the outer when both races are coaxially centered and no measurable force is applied. Free endplay depends on total curvature and contact angle, as shown in Fig. 21.3b, and can be written as

$$c_e = 2c_r \sin \beta_f \quad (21.10)$$

The variation of free contact angle and endplay with the ratio $c_d/2d$ is shown in Fig. 21.4 for four values of the total conformity ratio normally found in single-row ball bearings. Eliminating β_f in Eqs. (21.9) and (21.10) enables the following relationships between free endplay and diametral clearance to be established:

$$c_d = 2c_r - \left[(2c_r)^2 - c_e^2 \right]^{1/2}$$

$$c_e = (4c_r c_d - c_d^2)^{1/2}$$

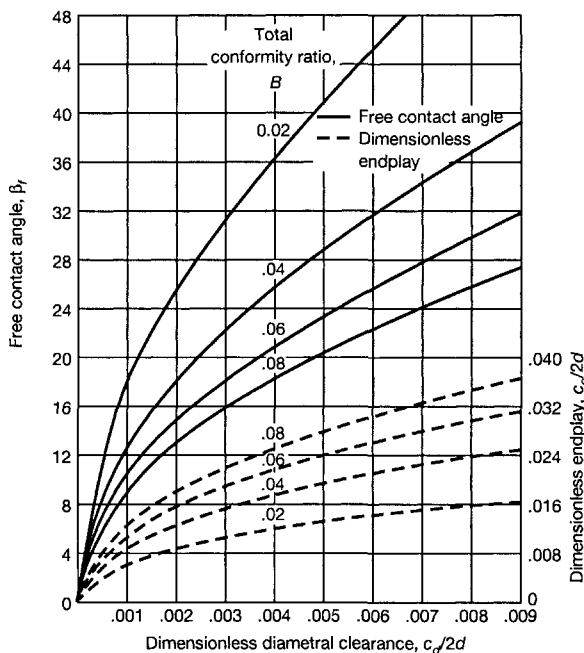


Figure 21.4: Free contact angle and endplay as a function of $c_d/2d$ for four values of total conformity. [From Hamrock and Anderson (1983)].

Shoulder Height

The shoulder height of ball bearings is illustrated in Fig. 21.5. Shoulder height, or race depth, is the depth of the race groove measured from the shoulder to the bottom of the groove and is denoted by s_h in Fig. 21.5. From this figure the equation defining the shoulder height can be written as

$$s_h = r(1 - \cos \theta_s) \quad (21.11)$$

The maximum possible diametral clearance for complete retention of the ball-race contact within the race under zero thrust load is given by the condition $(\beta_f)_{\max} = \theta_s$. Making use of Eqs. (21.9) and (21.11) gives

$$(c_d)_{\max} = \frac{2c_r s_h}{r}$$

Curvature Sum and Difference

The undeformed geometry of contacting solids in a ball bearing can be represented by two ellipsoids, as discussed in Sec. 17.2. This section applies the

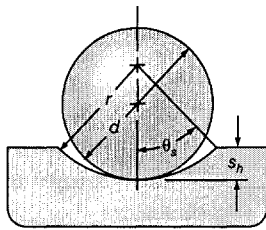


Figure 21.5: Shoulder height in ball bearing. [From Hamrock and Anderson (1983)].

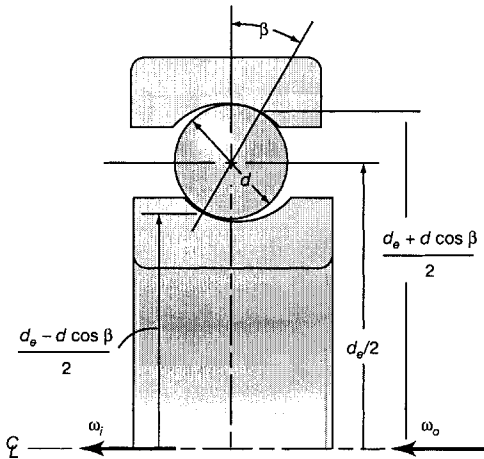


Figure 21.6: Cross section of ball bearing. [From Hamrock and Anderson (1983)].

information provided there to a ball bearing.

A cross section of a ball bearing operating at a contact angle β is shown in Fig. 21.6. Equivalent radii of curvature for both inner- and outer-race contacts in, and normal to, the direction of rolling can be calculated from this figure. The radii of curvature for the *ball-inner-race contact* are

$$r_{ax} = r_{ay} = \frac{d}{2} \quad (21.12)$$

$$r_{bx} = \frac{d_e - d \cos \beta}{2 \cos \beta} \quad (21.13)$$

$$r_{by} = -R_{r,i}d = -r_i \quad (21.14)$$

The radii of curvature for the *ball-outer-race contact* are

$$r_{ax} = r_{ay} = \frac{d}{2} \quad (21.15)$$

$$r_{bx} = -\frac{d_e + d \cos \beta}{2 \cos \beta} \quad (21.16)$$

$$r_{by} = -R_{r,o}d = -r_o \quad (21.17)$$

In Eqs. (21.13) and (21.16), β is used instead of β_f because these equations are also valid when a load is applied to the contact. By setting $\beta = 0^\circ$, Eqs. (21.12) to (21.17) are equally valid for radial ball bearings. For thrust ball bearings $r_{bx} = \infty$, and the other radii are defined as given in the preceding equations.

From the preceding radius-of-curvature expressions and Eq. (17.4) and (17.5) for the *ball-inner-race contact*

$$R_{x,i} = \frac{d(d_e - d \cos \beta)}{2d_e} \quad (21.18)$$

$$R_{y,i} = \frac{R_{r,i}d}{2R_{r,i} - 1} \quad (21.19)$$

and for the *ball-outer-race contact*

$$R_{x,o} = \frac{d(d_e + d \cos \beta)}{2d_e} \quad (21.20)$$

$$R_{y,o} = \frac{R_{r,o}d}{2R_{r,o} - 1} \quad (21.21)$$

Substituting these equations into Eqs. (17.2) and (17.3) enables the curvature sum and difference to be obtained.

21.4.2 Geometry of Roller Bearings

The equations developed for the pitch diameter d_e and diametral clearance c_d for ball bearings, Eqs. (21.1) and (21.2), respectively, are directly applicable for roller bearings.

Crowning

High stresses at the edges of the rollers in cylindrical roller bearings are usually prevented by crowning the rollers as shown in Fig. 21.7. A fully crowned roller is shown in Fig. 21.7a and a partially crowned roller in Fig. 21.7b. The crown curvature is greatly exaggerated for clarity. The crowning of rollers also gives the bearing protection against the effects of slight misalignment. For cylindrical rollers $r_r/d \approx 10^2$. In contrast, for spherical rollers in spherical roller bearings, as shown in Fig. 21.7a, $r_r/d \approx 4$. Observe in Fig. 21.7 that the roller effective length ℓ_t is the length presumed to be in contact with the races under loading. Generally, the roller effective length can be written as

$$\ell_\ell = \ell_t - 2r_c$$

where r_c is the roller corner radius or the grinding undercut, whichever is larger.

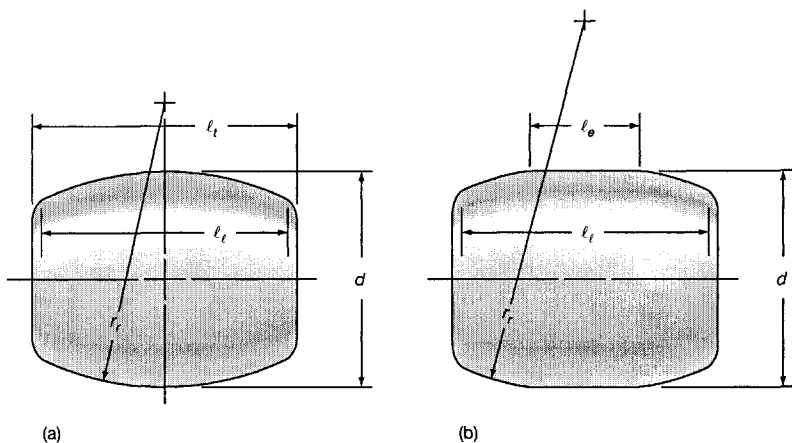


Figure 21.7: (a) Spherical roller (fully crowned) and (b) cylindrical roller (partially crowned). [From Hamrock and Anderson (1983)].

Race Conformity

Race conformity applies to roller bearings much as it applies to ball bearings. It is a measure of the geometrical conformity of the race and the roller. Figure 21.8 shows a cross section of a spherical roller bearing. From this figure the race conformity can be written as

$$R_r = \frac{r}{2r_r}$$

In this equation if R_r and r are subscripted with i or o , the race conformity values are for the inner- or outer-race contacts.

Free Endplay and Contact Angle

Cylindrical roller bearings have a contact angle of zero and may take thrust load only by virtue of axial flanges. Tapered-roller bearings must be subjected to a thrust load or the inner and outer races (the cone and the cup) will not remain assembled; therefore, tapered-roller bearings do not exhibit free diametral play. Radial spherical roller bearings are, however, normally assembled with free diametral play and hence exhibit free endplay. The diametral play c_d for a spherical roller bearing is the same as that obtained for ball bearings as expressed in Eq. (21.2). This diametral play as well as endplay is shown in Fig. 21.9 for a spherical roller bearing. From this figure

$$r_o \cos \beta = \left(r_o - \frac{c_d}{2} \right) \cos \gamma_d$$

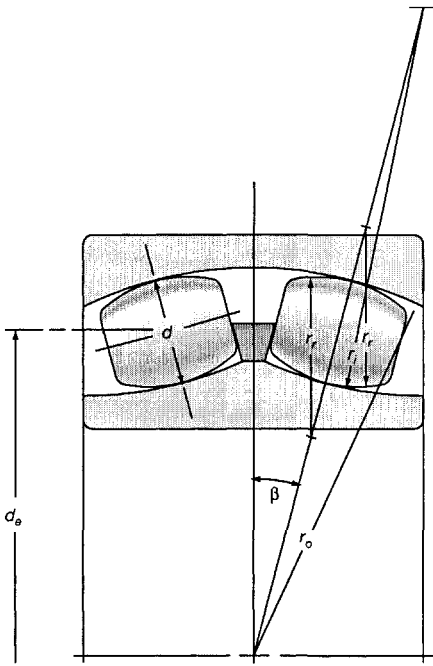


Figure 21.8: Geometry of spherical roller bearing. [From Hamrock and Anderson (1983)].

or

$$\beta = \cos^{-1} \left[\left(1 - \frac{c_d}{2r_o} \right) \cos \gamma_d \right]$$

Also from Fig. 21.9 the free endplay can be written as

$$c_e = 2r_o (\sin \beta - \sin \gamma_d) + c_d \sin \gamma_d$$

Curvature Sum and Difference

The same procedure used for ball bearings will be used for defining the curvature sum and difference for roller bearings. For spherical roller bearings, as shown in Fig. 21.8, the radii of curvature for the *roller-inner-race contact* can be written as

$$r_{ax} = \frac{d}{2}$$

$$r_{ay} = \frac{r_i}{2R_{r,i}} = r_r$$

$$r_{bx} = \frac{d_e - d \cos \beta}{2 \cos \beta}$$

$$r_{by} = -2R_{r,i}r_r = -r_i$$

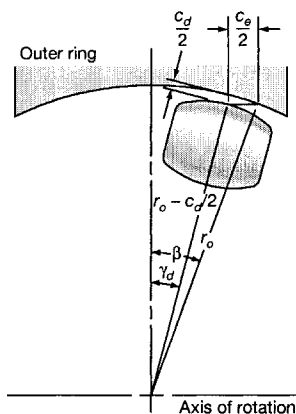


Figure 21.9: Schematic diagram of spherical roller bearing, showing diametral play and end-play. [From Hamrock and Anderson (1983)].

and the radii of curvature for the *roller-outer-race contact* can be written as

$$r_{ax} = \frac{d}{2}$$

$$r_{ay} = \frac{r_o}{2R_{r,o}} = r_r$$

$$r_{bx} = -\frac{d_e + d \cos \beta}{2 \cos \beta}$$

$$r_{by} = -2R_{r,o}r_r = -r_o$$

Once the radii of curvature for the respective contact conditions are known, the curvature sum and difference can be written directly from Eqs. (17.2) and (17.3). Furthermore, the radius-of-curvature expressions R_x and R_y for spherical roller bearings can be written for the *roller-inner-race contact* as

$$R_x = \frac{d(d_e - d \cos \beta)}{2d_e} \quad (21.22)$$

$$R_y = \frac{r_i r_r}{r_i - r_r} \quad (21.23)$$

and for the *roller-outer-race contact* as

$$R_x = \frac{d(d_e + d \cos \beta)}{2d_e} \quad (21.24)$$

$$R_y = \frac{r_o r_r}{r_o - r_r} \quad (21.25)$$

Substituting these equations into Eqs. (17.2) and (17.3) enables the curvature sum and difference to be obtained.

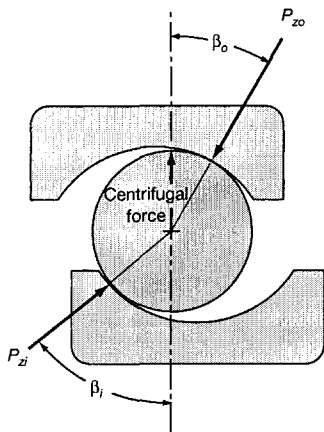


Figure 21.10: Contact angles in ball bearing at appreciable speeds. [From Hamrock and Anderson (1983)].

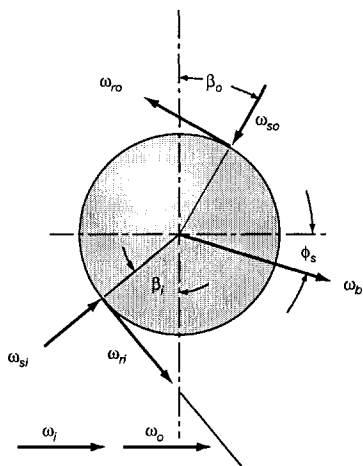


Figure 21.11: Angular velocities of ball. [From Hamrock and Anderson (1983)].

21.5 Kinematics

The relative motions of the separator, the balls or rollers, and the races of rolling-element bearings are important to understanding their performance. The relative velocities in a ball bearing are somewhat more complex than those in roller bearings, the latter being analogous to the specialized case of a zero- or fixed-contact-angle ball bearing. For that reason the ball bearing is used as an example here to develop approximate expressions for relative velocities. These expressions are useful for rapid but reasonably accurate calculation of elastohydrodynamic film thickness, which can be used with surface roughnesses to calculate the lubrication life factor.

The precise calculation of relative velocities in a ball bearing in which speed or centrifugal force effects, contact deformations, and elastohydrodynamic traction effects are considered requires a large computer to numerically solve the relevant equations. Refer to the growing body of computer codes discussed in Sec. 21.11 for precise calculations of bearing performance. Such a treatment is beyond the scope of this section. However, approximate expressions that yield answers with accuracies satisfactory for many situations are available.

When a ball bearing operates at high speeds, the centrifugal force acting on the ball creates a divergency of the inner- and outer-race contact angles, as shown in Fig. 21.10, in order to maintain force equilibrium on the ball. For the most general case of rolling and spinning at both inner- and outer-race contacts, the rolling and spinning velocities of the ball are as shown in Fig. 21.11.

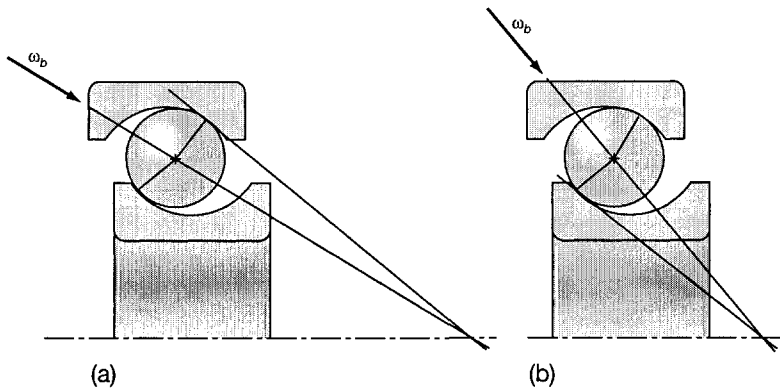


Figure 21.12: Ball spin axis orientations for (a) outer- and (b) inner-race control. [From Hamrock and Anderson (1983)].

The equations for ball and separator angular velocity for all combinations of inner- and outer-race rotation were developed by Jones (1964). Without introducing additional relationships to describe the elastohydrodynamic conditions at both ball-race contacts, however, the ball-spin axis orientation angle ϕ_s cannot be obtained. As mentioned, this requires a lengthy numerical solution except for the two extreme cases of outer- or inner-race control. These are illustrated in Fig. 21.12.

Race control assumes that pure rolling occurs at the controlling race, with all the ball spin occurring at the other race contact. The orientation of the ball rotation axis can then be easily determined from bearing geometry. Race control probably occurs only in dry bearings or dry-film-lubricated bearings where Coulomb friction conditions exist in the ball-race contact ellipses. The spin-resisting moment will always be greater at one of the race contacts. Pure rolling will occur at the race contact with the higher magnitude spin-resisting moment. This is usually the inner race at low speeds and the outer race at high speeds.

In oil-lubricated bearings in which elastohydrodynamic films exist in both ball-race contacts, rolling with spin occurs at both contacts. Therefore, precise ball motions can only be determined through a computer analysis. The situation can be approximated with a reasonable degree of accuracy, however, by assuming that the ball rolling axis is normal to the line drawn through the centers of the two ball-race contacts. This is shown in Fig. 21.6.

The angular velocity of the separator or ball set ω_c about the shaft axis can be shown to be (Anderson, 1970)

$$\omega_c = \frac{(v_i + v_o)/2}{d_e/2} = \frac{1}{2} \left[\omega_i \left(1 - \frac{d \cos \beta}{d_e} \right) + \omega_o \left(1 + \frac{d \cos \beta}{d_e} \right) \right] \quad (21.26)$$

where v_i and v_o are the linear velocities of the inner and outer contacts. The angular velocity of a ball about its own axis ω_b , assuming no spin, is

$$\omega_b = \frac{v_i - v_o}{d} = \frac{d_e}{2d} \left[\omega_i \left(1 - \frac{d \cos \beta}{d_e} \right) - \omega_o \left(1 + \frac{d \cos \beta}{d_e} \right) \right] \quad (21.27)$$

It is convenient, for calculating the velocities of the ball-race contacts, which are required for calculating elastohydrodynamic film thicknesses, to use a co-ordinate system that rotates at ω_c . This fixes the ball-race contacts relative to the observer. In the rotating coordinate system the angular velocities of the inner and outer races become

$$\omega_{ri} = \omega_i - \omega_c = \frac{\omega_i - \omega_o}{2} \left(1 + \frac{d \cos \beta}{d_e} \right)$$

$$\omega_{ro} = \omega_o - \omega_c = \frac{\omega_o - \omega_i}{2} \left(1 - \frac{d \cos \beta}{d_e} \right)$$

The surface velocities entering the *ball-inner-race contact* for pure rolling are

$$u_{ai} = u_{bi} = \frac{\omega_{ri} (d_e - d \cos \beta)}{2} \quad (21.28)$$

or

$$u_{ai} = u_{bi} = \frac{d_e (\omega_i - \omega_o)}{4} \left(1 - \frac{d^2 \cos^2 \beta}{d_e^2} \right) \quad (21.29)$$

and those at the *ball-outer-race contact* are

$$u_{ao} = u_{bo} = \frac{\omega_{ro} (d_e + d \cos \beta)}{2}$$

or

$$u_{ao} = u_{bo} = \frac{d_e (\omega_o - \omega_i)}{4} \left(1 - \frac{d^2 \cos^2 \beta}{d_e^2} \right) \quad (21.30)$$

Thus,

$$|u_{ai}| = |u_{ao}|$$

For a cylindrical roller bearing $\beta = 0^\circ$, and Eqs. (21.26), (21.27), (21.29), and (21.30) become, if d is roller diameter,

$$\begin{aligned} \omega_c &= \frac{1}{2} \left[\omega_i \left(1 - \frac{d}{d_e} \right) + \omega_o \left(1 + \frac{d}{d_e} \right) \right] \\ \omega_b &= \frac{d_e}{2d} \left[\omega_i \left(1 - \frac{d}{d_e} \right) + \omega_o \left(1 + \frac{d}{d_e} \right) \right] \\ u_{ai} &= u_{bi} = \frac{d_e (\omega_i - \omega_o)}{4} \left(1 - \frac{d^2}{d_e^2} \right) \\ u_{ao} &= u_{bo} = \frac{d_e (\omega_o - \omega_i)}{4} \left(1 - \frac{d^2}{d_e^2} \right) \end{aligned} \quad (21.31)$$

Equations directly analogous to those for a ball bearing can be used for a tapered-roller bearing if d is the average diameter of the tapered roller, d_e is the diameter at which the geometric center of the rollers is located, and β is the angle as shown in Fig. 21.13.

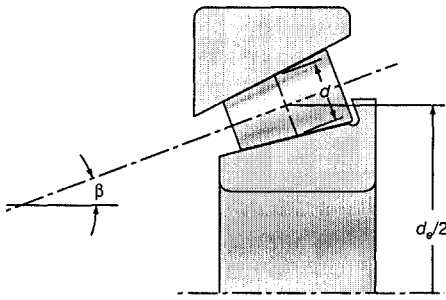


Figure 21.13: Simplified geometry for tapered-roller bearing. [From Hamrock and Anderson (1983)].

21.6 Separators

Ball and rolling-element bearing separators (sometimes called “cages” or “retainers”) are bearing components that, although they never carry load, are capable of exerting a vital influence on bearing efficiency. In a bearing without a separator the rolling elements contact each other during operation and in so doing experience severe sliding and friction. The primary functions of a separator are to maintain the proper distance between the rolling elements and to ensure proper load distribution and balance within the bearing. Another function of the separator is to maintain control of the rolling elements in such a manner as to produce the least possible friction through sliding contact. Furthermore, a separator is necessary for several types of bearing to prevent the rolling elements from falling out of the bearing during handling. Most separator troubles occur from improper mounting, misaligned bearings, or improper (inadequate or excessive) clearance in the rolling-element pocket.

The materials used for separators vary according to the type of bearing and the application, in ball bearings and some sizes of roller bearing the most common type of separator is made from two strips of carbon steel that are pressed and riveted together. Called “ribbon separators,” they are the least expensive separators to manufacture and are entirely suitable for many applications. They are also lightweight and usually require little space.

The design and construction of angular-contact ball bearings allow the use of a one-piece separator. The simplicity and inherent strength of one-piece separators permit their fabrication from many desirable materials. Reinforced phenolic and bronze are the two most commonly used materials. Bronze separators offer strength and low-friction characteristics and can be operated at temperatures to 230°C (450°F). Machined, silver-plated ferrous alloy separators are used in many demanding applications. Because reinforced cotton-base phenolic separators combine the advantages of low weight, strength, and non-galling properties, they are used for such high-speed applications as gyro bearings. Lightness and strength are particularly desirable in high-speed bearings, since the stresses increase with speed but may be greatly minimized by reducing separator weight. A limitation of phenolic separators, however, is that they

have an allowable maximum temperature of about 135°C (275°F).

21.7 Static Load Distribution

Since a simple analytical expression for the deformation in terms of load was defined in Sec. 17.5, it is possible to consider how the bearing load is distributed among the elements. Most rolling-element bearing applications involve steady state rotation of either the inner or outer race or both; however, the rotational speeds are usually not so great as to cause ball or roller centrifugal forces or gyroscopic moments of significant magnitudes. In analyzing the loading distribution on the rolling elements, it is usually satisfactory to ignore these effects in most applications. In this section the load deflection relationships for ball and roller bearings are given, along with radial and thrust load distributions of statically loaded rolling elements.

21.7.1 Load Deflection Relationships

For an elliptical conjunction the load deflection relationship given in Eq. (17.16) can be written as

$$w_z = K_{1.5} \delta_m^{3/2} \quad (21.32)$$

where

$$K_{1.5} = \pi k E' \left(\frac{2 \mathcal{E} R}{9 \mathcal{F}^3} \right)^{1/2} \quad (21.33)$$

Similarly, for a rectangular conjunction

$$w_z = K_1 \delta_m$$

where

$$K_1 = \frac{\pi \ell E'}{2 \left[\frac{2}{3} + \ln(4r_{ax}/b) + \ln(4r_{bx}/b) \right]} \quad (21.34)$$

and ℓ is the length of the rolling element. In general then,

$$w_z = K_j \delta_m^j \quad (21.35)$$

in which $j = 1.5$ for ball bearings and 1.0 for roller bearings. The total normal approach between two races separated by a rolling element is the sum of the deformations under load between the rolling element and both races. Therefore,

$$\delta_m = \delta_{mo} + \delta_{mi} \quad (21.36)$$

where

$$\delta_{mo} = \left(\frac{w_z}{(K_j)_o} \right)^{1/j} \quad (21.37)$$

$$\delta_{mi} = \left(\frac{w_z}{(K_j)_i} \right)^{1/j} \quad (21.38)$$

Substituting Eqs. (21.36) to (21.38) into Eq. (21.35) gives

$$K_j = \frac{1}{\left\{ [1/(K_j)_o]^{1/j} + [1/(K_j)_i]^{1/j} \right\}^j} \quad (21.39)$$

Recall that $(K_j)_o$ and $(K_j)_i$ are defined by Eq. (21.33) and (21.34) for an elliptical and a rectangular conjunction, respectively. These equations show that $(K_j)_o$ and $(K_j)_i$ are functions only of the contact geometry and the material properties. The radial and thrust load analyses are presented in the following two sections and are directly applicable for radially loaded ball and roller bearings and thrust-loaded ball bearings.

21.7.2 Radially Loaded Ball and Roller Bearings

A radially loaded rolling element with radial clearance $c_d/2$ is shown in Fig. 21.14. In the concentric position shown in Fig. 21.14a a uniform radial clearance between the rolling element and the races of $c_d/2$ is evident. The application of an arbitrarily small radial load to the shaft causes the inner race to move a distance $c_d/2$ before contact is made between a rolling element located on the load line and the inner and outer races. At any angle there will still be a radial clearance c that, if c_d is small in relation to the radius of the tracks, can be expressed with adequate accuracy by

$$c = (1 - \cos \psi) \frac{c_d}{2}$$

On the load line where $\psi = 0^\circ$, the clearance is zero; but when $\psi = 90^\circ$, the clearance retains its initial value of $c_d/2$.

The application of further load will cause elastic deformation of the balls and elimination of clearance around an arc $2\psi_\ell$. If the interference or total elastic compression on the load line is $\delta_{\max}$, the corresponding elastic compression of the ball δ_ψ , along a radius at angle ψ to the load line will be given by

$$\delta_\psi = (\delta_{\max} \cos \psi - c) = \left(\delta_{\max} + \frac{c_d}{2} \right) \cos \psi - \frac{c_d}{2}$$

This assumes that the races are rigid. Now it is clear from Fig. 21.14c that $\delta_{\max} + c_d/2$ represents the total relative radial displacements of the inner and outer races. Hence,

$$\delta_\psi = \delta_m \cos \psi - \frac{c_d}{2} \quad (21.40)$$

The relationship between load and elastic compression along the radius at angle ψ to the load vector is given by Eq. (21.35) as

$$w_\psi = K_j \delta_\psi^j$$

Substituting Eq. (21.40) into this equation gives

$$w_\psi = K_j \left(\delta_m \cos \psi - \frac{c_d}{2} \right)^j$$

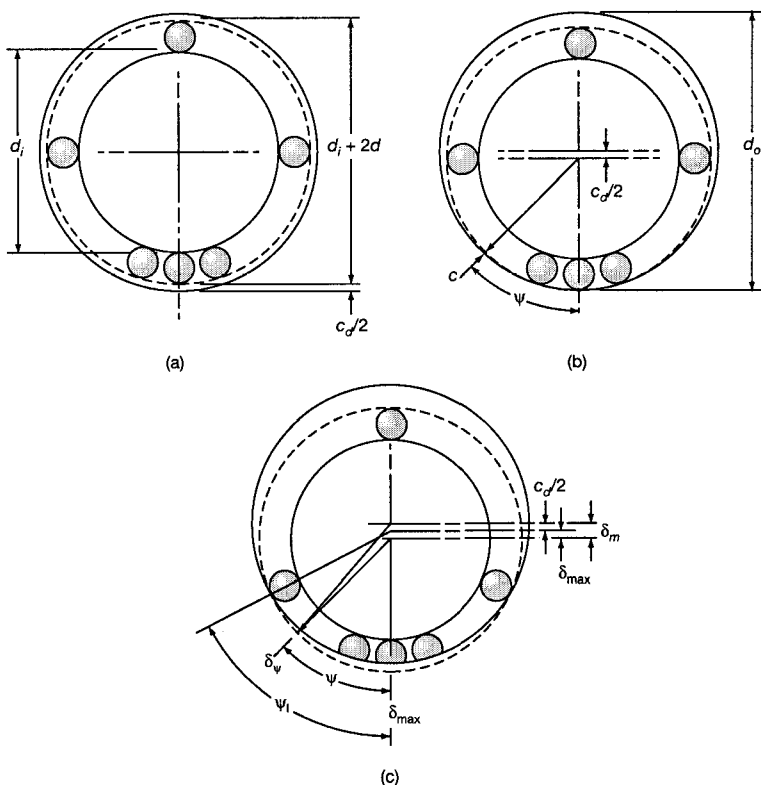


Figure 21.14: Radially loaded rolling-element bearing. (a) Concentric arrangement; (b) initial contact; (c) interference. [From Hamrock and Anderson (1983)].

For static equilibrium the applied load must equal the sum of the components of the rolling-element loads parallel to the direction of the applied load.

$$w_t = \sum w_\psi \cos \psi$$

Therefore,

$$w_t = K_j \sum \left(\delta_m \cos \psi - \frac{c_d}{2} \right)^j \cos \psi \quad (21.41)$$

The angular extent of the bearing arc $2\psi_\ell$ in which the rolling elements are loaded is obtained by setting the root expression in Eq. (21.41) equal to zero and solving for ψ :

$$\psi_\ell = \cos^{-1} \frac{c_d}{2\delta_m}$$

The summation in Eq. (21.41) applies only to the angular extent of the loaded region. This equation can be written for a roller bearing as

$$w_t = \left(\psi_\ell - \frac{c_d}{2\delta} \sin \psi_\ell \right) \frac{n K_1 \delta_m}{2\pi} \quad (21.42)$$

and similarly in integral form for a ball bearing as

$$w_t = \frac{n}{P} K_{1.5} \delta_m^{3/2} \int_0^{\psi_\ell} \left(\cos \psi - \frac{c_d}{2\delta} \right)^{3/2} \cos \psi \, d\psi$$

The integral in the equation can be reduced to a standard elliptic integral by the hypergeometric series and the beta function. If the integral is numerically evaluated directly, the following approximate expression is derived:

$$\int_0^{\psi_\ell} \left(\cos \psi - \frac{c_d}{2\delta_m} \right)^{3/2} \cos \psi \, d\psi = 2.491 \left\{ \left[1 + \left(\frac{c_d/2\delta_m - 1}{1.23} \right)^2 \right]^{1/2} - 1 \right\}$$

This approximate expression fits the exact numerical solution to within $\pm 2\%$ for a complete range of $c_d/2\delta_m$.

The load carried by the most heavily loaded rolling element is obtained by substituting $\psi = 0^\circ$ in Eq. (21.41) and dropping the summation sign.

$$(w_z)_{\max} = K_j \delta_m^j \left(1 - \frac{c_d}{2\delta_m} \right)^j \quad (21.43)$$

Dividing this maximum load by the total radial load for a roller bearing [Eq. (21.42)] gives

$$w_z = \frac{\left(\psi_\ell - \frac{c_d}{2\delta_m} \sin \psi_\ell \right) \frac{n (w_z)_{\max}}{2\pi}}{1 - c_d/2\delta_m} \quad (21.44)$$

and similarly for a ball bearing

$$w_z = \frac{n (w_z)_{\max}}{Z_w} \quad (21.45)$$

where

$$Z_w = \frac{\pi (1 - c_d/2\delta_m)^{3/2}}{2.491 \left\{ \left[1 + \left(\frac{1 - c_d/2\delta_m}{1.23} \right)^2 \right]^{1/2} - 1 \right\}} \quad (21.46)$$

For roller bearings when the diametral clearance c_d is zero, Eq. (21.44) gives

$$w_z = \frac{n (w_z)_{\max}}{4} \quad (21.47)$$

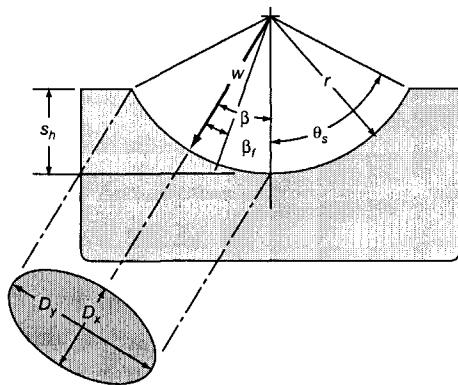


Figure 21.15: Contact ellipse in bearing race under load. [From Hamrock and Anderson (1983)].

For ball bearings when the diametral clearance c_d is zero, the value of Z_w in Eq. (21.45) becomes 4.37. This is the value derived by Stribeck (1901) for ball bearings of zero diametral clearance. The approach used by Stribeck was to evaluate the finite summation for various numbers of balls. He then derived the celebrated Stribeck equation for static load-carrying capacity by writing the more conservative value of 5 for the theoretical value of 4.37:

$$w_z = \frac{n(w_z)_{\max}}{5} \quad (21.48)$$

In using Eq. (21.48) remember that Z_w was considered to be a constant and that the effects of clearance and applied load on load distribution were not taken into account. These effects were, however, considered in obtaining Eq. (21.45).

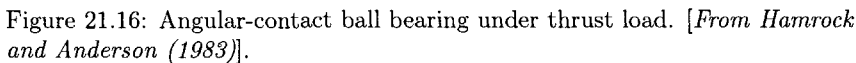
21.7.3 Thrust-Loaded Ball Bearing

The static thrust load-carrying capacity of a ball bearing may be defined as the maximum thrust load that the bearing can endure before the contact ellipse approaches a race shoulder, as shown in Fig. 21.15, or as the load at which the allowable mean compressive stress is reached, whichever is smaller. Both the limiting shoulder height and the mean compressive stress must be calculated to find the static thrust load-carrying capacity.

Each ball is subjected to an identical thrust component w_t/n , where w_t is the total thrust load. The initial contact angle before the application of a thrust load is denoted by β_f . Under load the normal ball thrust load w_t acts at the contact angle β and is written as

$$w_z = \frac{w_t}{n \sin \beta} \quad (21.49)$$

A cross section through an angular-contact bearing under a thrust load w_t is shown in Fig. 21.16. From this figure the contact angle after the thrust load


$$\beta = \cos^{-1} \frac{c_r - c_d/2}{c_r + \delta_m} \quad (21.50)$$
$$\delta_m = c_r \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)$$

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$$w = K_{1.5} c_r^{3/2} \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)^{3/2} \quad (21.52)$$

where Eq. (21.32) for $K_{1.5}$ is replaced by Eq. (21.51) and k , $\mathcal{E}$, $\mathcal{F}$ are given by Eq. (17.29), (17.30), and (17.32), respectively.

From Eq. (21.49) and (21.52),

$$\frac{w_t}{n \sin \beta} = w_z$$

or

$$\frac{w_t}{n K_j c_r^{3/2}} = \sin \beta \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)^{3/2} \quad (21.53)$$

This equation can be solved numerically by the Newton-Raphson method. The iterative equation to be satisfied is

$$\beta' - \beta = \frac{\frac{w_t}{n K_{1.5} c_r^{3/2}} - \sin \beta \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)^{3/2}}{\cos \beta \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)^{3/2} + \frac{3}{2} \cos \beta_f \tan^2 \beta \left(\frac{\cos \beta_f}{\cos \beta} - 1 \right)^{1/2}} \quad (21.54)$$

In this equation convergence is satisfied when $\beta' - \beta$ becomes essentially zero.

When a thrust load is applied, the shoulder height limits the axial deformation, which can occur before the pressure-contact ellipse reaches the shoulder. As long as the following inequality is satisfied, the contact ellipse will not exceed this limit:

$$\theta_s > \beta + \sin^{-1} \frac{D_y}{R_r d}$$

From Fig. 21.5 and Eq. (21.11) the angle θ_s used to define the shoulder height can be written as

$$\theta_s = \cos^{-1} \left(1 - \frac{s_h}{R_r d} \right)$$

From Fig. 21.3 the axial deflection δ_t corresponding to a thrust load can be written as

$$\delta_t = (c_r + \delta) \sin \beta - c_r \sin \beta_f \quad (21.55)$$

Substituting Eq. (21.51) into (21.55) gives

$$\delta_t = \frac{c_r \sin(\beta - \beta_f)}{\cos \beta}$$

Once β has been determined from Eq. (21.54) and β_f from Eq. (21.7), the relationship for δ_t can be easily evaluated.

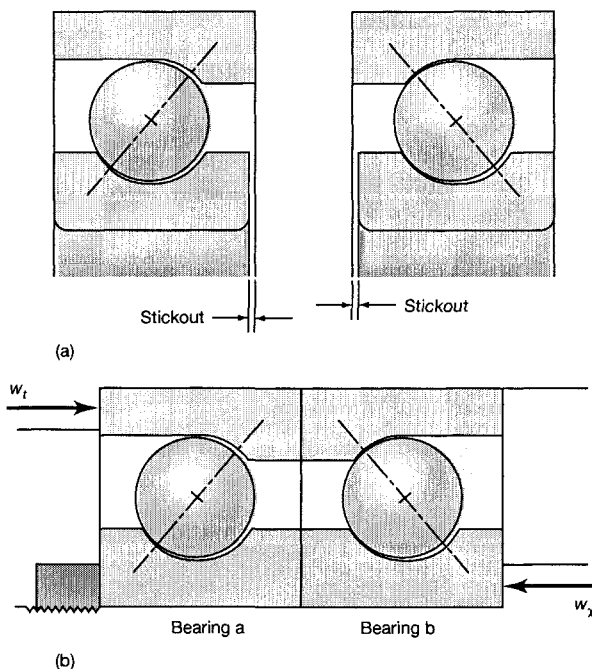


Figure 21.17: Angular-contact bearings in back-to-back arrangement, shown (a) individually as manufactured and (b) as mounted with preload. [From Hamrock and Anderson (1983)].

21.7.4 Preloading

The use of angular-contact bearings as duplex pairs preloaded against each other is discussed in Sec. 21.3.1. As shown in Table 21.2, duplex bearing pairs are used in either back-to-back or face-to-face arrangements. Such bearings are usually preloaded against each other by providing what is called “stickout” in the manufacture of the bearing. This is illustrated in Fig. 21.17 for a bearing pair used in a back-to-back arrangement. The magnitude of the stickout and the bearing design determine the level of preload on each bearing when the bearings are clamped together as in Fig. 21.17. The magnitude of preload and the load deflection characteristics for a given bearing pair can be calculated by using Eqs. (21.7), (21.32), (21.49), and (21.51) to (21.53).

The relationships of initial preload, system load, and final load for bearings a and b are shown in Fig. 21.18. The load deflection curve follows the relationship $\delta_m = Kw^{2/3}$. When a system thrust load w_t is imposed on the bearing pairs, the magnitude of load on bearing b increases while that on bearing a

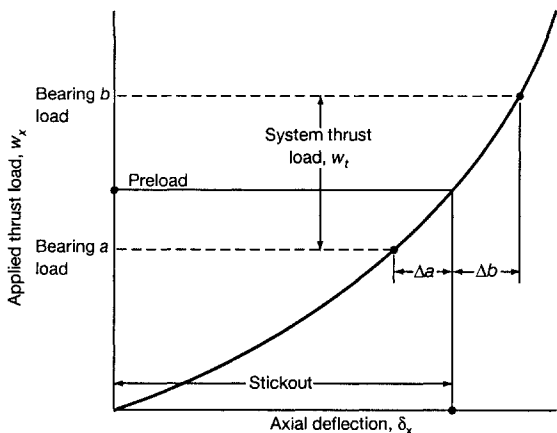


Figure 21.18: Thrust load - axial deflection curve for typical ball bearing. [From Hamrock and Anderson (1983)].

decreases until the difference equals the system load. The physical situation demands that the change in each bearing deflection be the same ($\Delta a = \Delta b$ in Fig. 21.18). The increments in bearing load, however, are not the same. This is important because it always requires a system thrust load far greater than twice the preload before one bearing becomes unloaded. Prevention of bearing unloading, which can result in skidding and early failure, is an objective of preloading.

21.8 Rolling Friction and Friction Losses

21.8.1 Rolling Friction

The concepts of rolling friction are important generally in understanding the behavior of machine elements in rolling contact and particularly because rolling friction influences the overall behavior of rolling element bearings. The theories of Reynolds (1886) and Heathcote (1921) attempted to explain rolling friction in terms of the energy required to overcome the interfacial slip that occurs because of the curved shape of the contact area. As shown in Fig. 21.19, the ball rolls about the Y axis and makes contact with the groove from a to b . If the groove is fixed, then for zero slip over the contact area no point within the area should have a velocity in the rolling direction. The surface of the contact area is curved, however, so that points a and b are at different radii from the Y axis than are points c and d . For a rigid ball, points a and b must have different velocities with respect to the Y axis than do points c and d because the velocity of any point on the ball relative to the Y axis equals the angular velocity times the radius from the Y axis. Slip must occur at various points over the contact area unless the body is so elastic that yielding can take place in the contact area to prevent this interfacial slip. The theories of Reynolds and

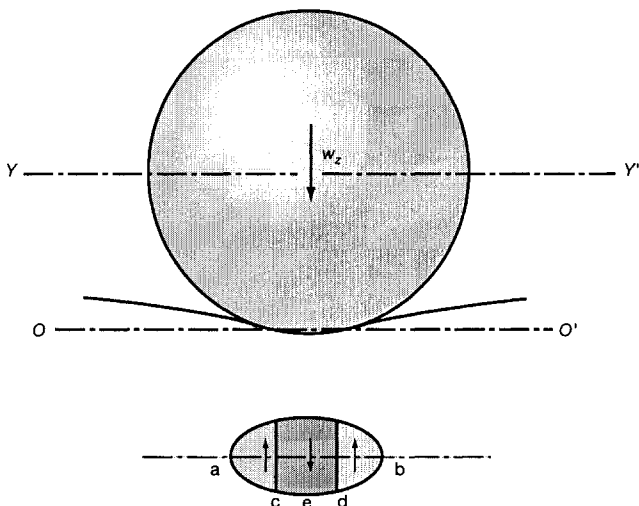


Figure 21.19: Differential slip due to curvature of contact ellipse. [From Hamrock and Anderson (1983)].

later Heathcote assumed that this interfacial slip took place and that the forces required to make a ball roll were those required to overcome the friction due to this interfacial slip. In the contact area rolling without slip will occur at a specific radius from the Y axis. Where the radius is greater than this radius to the rolling point, slip will occur in one direction; where it is less, slip will occur in the other direction. In Fig. 21.19 the lines at points c and d represent the approximate location of the rolling bands, and the arrows shown in the three portions of the contact area represent the directions of interfacial slip when the ball is rolling into the paper.

The location of the two rolling bands relative to the axis of the contact ellipse can be obtained by summing the loads acting on the ball in the rolling direction. In Fig. 21.20 these are

$$2w_{x,b} - w_{x,a} = \mu_r w_z$$

where μ_r is the coefficient of rolling friction. If a Hertzian ellipsoidal pressure distribution is assumed, the location of the rolling bands can be determined (Bisson and Anderson, 1964). For rolling with zero traction ($\mu_r = 0$), the result is

$$\frac{y_o}{D_y} \approx 0.174$$

where D_y is the diameter of the contact ellipse along the y axis [Eq. (17.14)].

In reality, all materials are elastic so that areas of no slip as well as areas of microslip exist within the contact as pointed out by Johnson and Tabor (1967-1968). Differential strains in the materials in contact will cause slip unless it is prevented by friction. In high-conformity contacts slip is likely to occur over most of the contact region, as can be seen in Fig. 21.21, which shows the nondimensional friction force as a function of the conformity parameter

$$\lambda_f = \frac{\pi D_y^2 E'}{4\mu r_{ax}^2 p_{\text{mean}}}$$

where μ is the coefficient of sliding friction and p_{mean} is the mean pressure of the contact ellipse. The limiting value of friction force w_x is

$$w_x = \frac{0.02\mu D_y^2 w_z}{r_{ax}^2}$$

A fraction λ_h of the elastic energy of compression in rolling is always lost because of hysteresis. The effect of hysteresis losses on rolling resistance has been studied by Tabor (1955). Tabor developed the following expression:

$$w_x = c_4 \lambda_h \frac{w_z D_y}{r_{ax}} \quad (21.56)$$

where c_4 is $1/3\pi$ for rectangular conjunctions and $3/32$ for elliptical conjunctions. Two hard-steel surfaces show a λ_h of about 1 percent.

Of far greater importance in contributing to frictional losses in rolling-element bearings, especially at high speeds, is ball spinning. Spinning as well as rolling takes place in one or both of the ball-race contacts of a ball bearing. The situation is shown schematically in Fig. 21.11.

High speeds cause a divergency of the contact angles β_i and β_o . In Fig. 21.12b the ball is rolling with inner-race control so that approximately pure rolling takes place at the inner-race contact. The rolling and spinning vectors ω_r and ω_s at the outer-race contact are shown in Fig. 21.11. The higher the ratio ω_s/ω_r , the higher the friction losses.

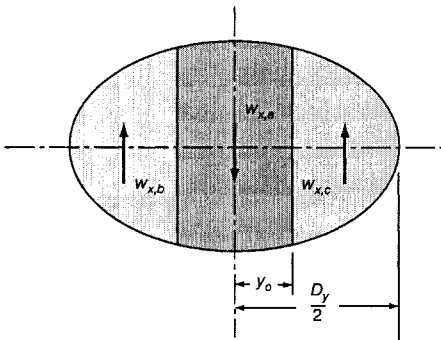


Figure 21.20: Load components in contact ellipse. [From Hamrock and Anderson (1983)].

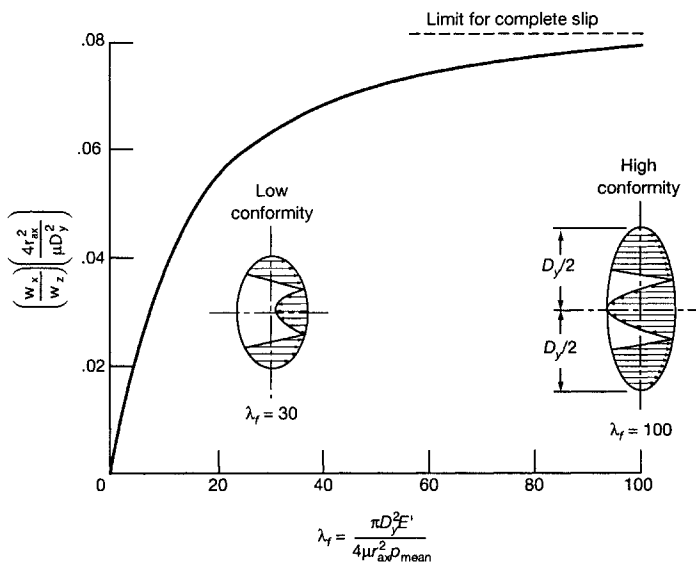


Figure 21.21: Frictional resistance of ball in conforming groove. [From Hamrock and Anderson (1983)].

21.8.2 Friction Losses

Some factors that affect the magnitude of friction losses in rolling-element bearings are

1. Bearing size
2. Bearing type
3. Bearing design
4. Load (magnitude and type, either thrust or radial)
5. Speed
6. Oil viscosity
7. Oil flow

Friction losses in a specific bearing consist of the following:

1. Sliding friction losses in the contacts between the rolling elements and the races. These losses include differential slip and slip due to ball spinning. They are complicated by the presence of elastohydrodynamic lubrication

films. The shearing of these films, which are extremely thin and contain oil whose viscosity is increased by orders of magnitude above its atmospheric pressure value, accounts for a significant fraction of the friction losses in rolling-element bearings.

2. Hysteresis losses due to the damping capacity of the race and ball materials.
3. Sliding friction losses between the separator and its locating race surface and between the separator pockets and the rolling elements.
4. Shearing of oil films between the bearing parts and oil churning losses caused by excess lubricant within the bearing.
5. Flinging of oil off the rotating parts of the bearing.

Because of the dominant role in bearing frictional losses played by the lubrication method, there are no quick and easy formulas for computing rolling-element bearing power loss. Friction coefficients for a particular bearing can vary by a factor of 5 depending on lubrication. A flood-lubricated bearing may consume five times the power of one that is merely wetted by oil/air mist lubrication.

The friction coefficients given by Palmgren (1959) are a rough but useful guide. These values were computed at a bearing load that will give a life of 1×10^9 revolutions for the the respective bearings. The friction coefficients for several different bearings are shown here:

Self-aligning ball	0.0010
Cylindrical roller with flange-guided short rollers	0.0011
Thrust ball	0.0013
Single-row, deep-groove ball	0.0015
Tapered and spherical roller with flange-guided rollers	0.0018
Needle roller	0.0045

All these friction coefficients are referenced to the radius of the bearing bore.

More accurate estimates of bearing power loss and temperature rise can be obtained by using one or more of the available computer codes that represent the basis for the current design methodology for rolling-element bearings. These are discussed in Anderson (1979) and Pirvics (1980) and briefly reviewed in Sec. 21.11.

21.9 Lubrication Systems

A liquid lubricant has several functions in a rolling-element bearing. It provides separating films between the bearing parts (elastohydrodynamic lubrication between the races and the rolling elements and hydrodynamic between the cage

or separator and its locating surface). It serves as a coolant if either circulated through the bearing to an external heat exchanger or simply brought into contact with the bearing housing and the machine casing. A circulating lubricant also serves to flush out wear debris and carry it to a filter where it can be removed from the system. Finally, it provides corrosion protection. The different methods of providing liquid lubricant to a bearing are each discussed here. Note that Chapter 4 presents lubricant properties, whereas this section explores effective means of lubrication.

21.9.1 Solid Lubrication

An increasing number of rolling-element bearings are lubricated with solid-film lubricants, usually in applications, such as extreme temperature or the vacuum of space, where conventional liquid lubricants are not suitable.

Success in cryogenic applications, where the bearing is cooled by the cryogenic fluid (liquid oxygen or hydrogen), has been achieved with transfer films of polytetrafluoroethylene (Scibbe, 1968). Bonded films of soft metals such as silver, gold, and lead applied by ion plating as extremely thin films (0.2 to 0.3 μm) have also been used (Todd and Bentall, 1978). Silver and lead in particular have found use in bearings that support the rotating anode in x-ray tubes. Extremely thin films are required in rolling-element bearings in order not to significantly alter the bearing internal geometry and in order to retain the basic mechanical properties of the substrate materials in the Hertzian contacts.

21.9.2 Liquid Lubrication

The great majority of rolling-element bearings are lubricated by liquids. The liquid in greases can be used, or liquid can be supplied to the bearing from either noncirculating or circulating systems.

Greases

The most common and probably least expensive mode of lubrication is grease lubrication; in the strictest sense a grease is not a liquid, but the liquid or fluid constituent in the grease is the lubricant. Greases consist of a fluid phase of either a petroleum or synthetic oil and a thickener. The most common thickeners are sodium-, calcium-, or lithium-based soaps, although thickeners of inorganic materials such as bentonite clay have been used in synthetic greases. Some discussion of the characteristics and temperature limits of greases is given in Bisson and Anderson (1964) and McCarthy (1973).

Greases are usually retained within the bearing by shields or seals that are an integral part of the assembled bearing. Since there is no recirculating fluid, grease-lubricated bearings must reject heat by conduction and convection and are therefore limited to maximum $d_b N_a$ (ball bearing bore diameter in millimeters times rotational speed in revolutions per minute) values of 0.25 to 0.4 million.

The proper grease for a particular application depends on the temperature, speed, and ambient pressure environment to which the bearing is exposed. McCarthy (1973) presents a comprehensive discussion useful in the selection of a grease; the bearing manufacturer can also recommend the most suitable grease and bearing type.

Nonrecirculating Liquid Lubrication Systems

At low to moderate speeds, where the use of grease lubrication is not suitable, other methods of supplying lubricant to the bearing can be used. These include splash or bath, wick, oil-ring, and oil-air mist lubrication. Felt wicks can be used to transport oil by capillary action from a nearby reservoir. Oil rings, which are driven by frictional contact with the rotating shaft, run partially immersed in an oil reservoir and feed oil mechanically to the shaft, which is adjacent to the bearing. The bearing may itself be partially immersed in an oil reservoir to splash-lubricate itself. All these methods require modest ambient temperatures and thermal conditions as well as speed conditions equivalent to a maximum $d_b N_a$ of about 0.5 million. The machinery must also remain in a fixed-gravity orientation.

Oil/air mist lubrication supplies atomized oil in an airstream to the bearing, where a reclassifier increases the droplet size, allowing it to condense on the bearing surfaces. Feed rates are low, and a portion of the oil flow escapes with the feed air to the atmosphere. Commercial oil/air mist generators are available for systems ranging from a single bearing to hundreds of bearings. Bearing friction losses and heat generation are low with mist lubrication, but ambient temperatures and cooling requirements must be moderate because oil/air mist systems provide minimal cooling. Many bearings, especially small bore bearings, are successfully operated at high speeds ($d_b N_a$ values to greater than 1 million) with oil/air mist lubrication.

Jet Lubrication

In applications where speed or heat rejection requirements are too high, jet lubrication is frequently used both to lubricate and to control bearing temperatures. A number of variables are critical to achieving not only satisfactory but near optimal performance and bearing operation. These include the placement of the nozzles, the number of nozzles, the jet velocity, the lubricant flow rate, and the scavenging of the lubricant from the bearing and its immediate vicinity. The importance of proper jet lubricating system design is shown by Matt and Gianotti (1966). Their results are summarized in Fig. 21.22.

Proper placement of the jets should take advantage of any natural pumping ability of the bearings. Figure 21.23 (Parker, 1980) illustrates jet lubrication of ball and tapered-roller bearings. Centrifugal forces aid in moving the oil through the bearing to cool and lubricate the elements. Directing jets at the radial gaps between the cage and the races achieves maximum oil penetration into the bearing interior. Miyakawa et al. (1972), Anderson et al. (1954),

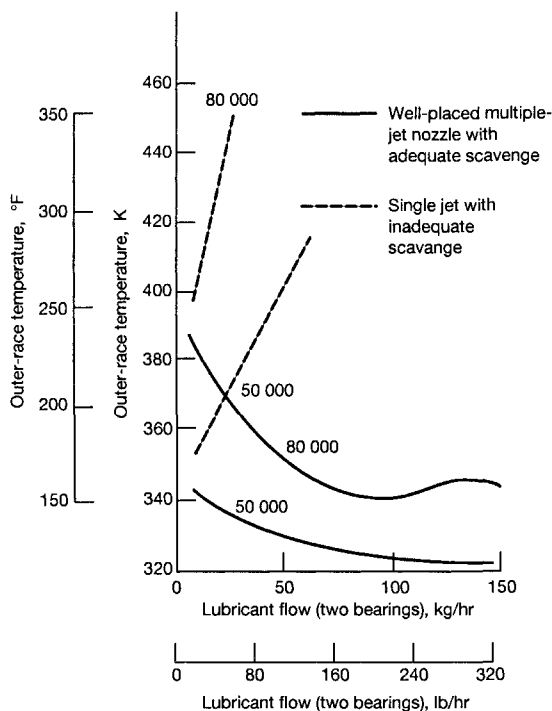


Figure 21.22: Effectiveness of proper jet lubrication. Test bearings: 20-mm-bore, angular-contact ball bearings; thrust load, 222 N (50 lbf). [From Matt and Gianotti (1966)].

Zaretsky et al. (1976), and Parker and Signer (1978) present useful data on how jet placement and velocity influence the lubrication of several types of bearing.

Underrace Lubrication

As bearing speeds increase, centrifugal effects become more predominant, making it increasingly difficult to lubricate and cool a bearing effectively. The jetted oil is thrown off the sides of the bearing rather than penetrating to the interior. At extremely high $d_b N_a$ values (2.4 million and higher), jet lubrication becomes ineffective. Increasing the flow rate only adds to heat generation through increased churning losses. Brown (1970) describes an underrace oiling system used in a turbofan engine for both ball and cylindrical roller bearings. Figure 21.24 illustrates the technique. The lubricant is directed radially under the bearings. Centrifugal effects assist in pumping oil out through the bearings via suitable slots and holes, which are made a part of both the shaft and the mounting system and the bearings themselves. Holes and slots are provided within the bearing to feed oil directly to the ball-race and cage-race contacts.

This lubrication technique has been thoroughly tested for large-bore ball

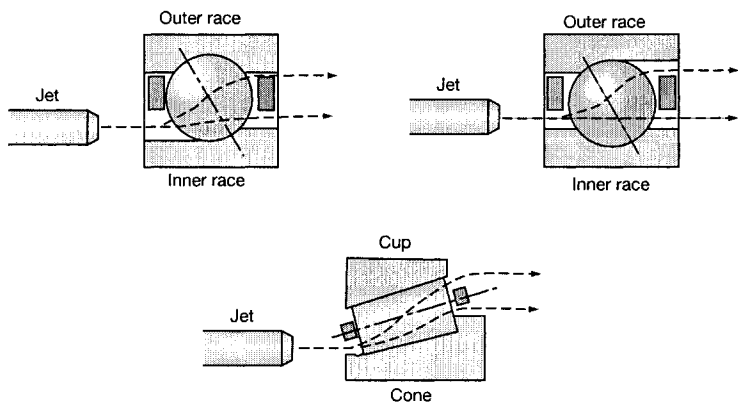


Figure 21.23: Placement of jets for ball bearings with relieved rings and tapered-roller bearings. [From Parker (1980)].

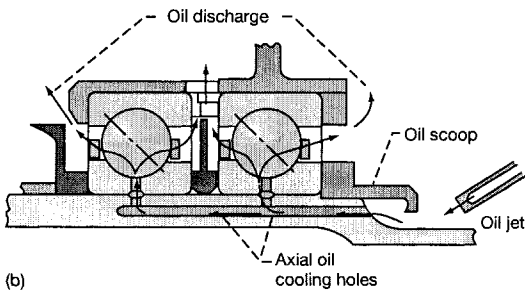
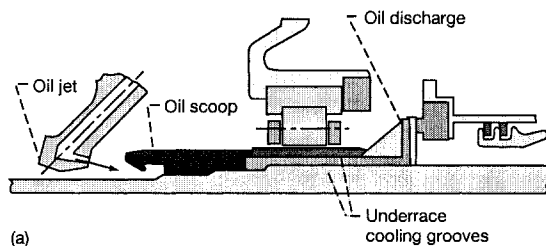


Figure 21.24: Under-race oiling system for main shaft bearings on turbofan engine. (a) Cylindrical roller bearing; (b) ball thrust bearing. [From Brown (1970)].

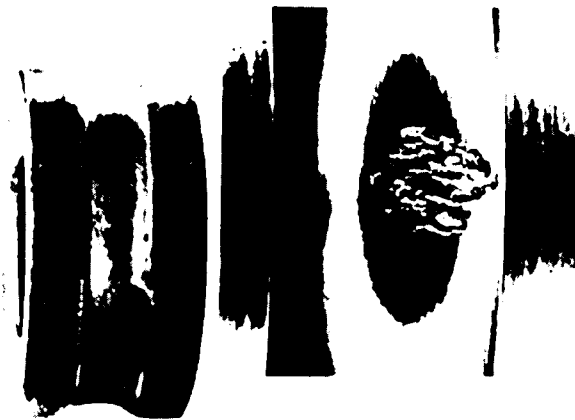


Figure 21.25: Typical fatigue spall. [From Hamrock and Anderson (1983)].

and roller bearings up to 3 million $d_b N_a$. Pertinent data are reported by Signer et al. (1974), Brown et al. (1977), Schuller (1979), and Signer and Schuller (1982).

21.10 Fatigue Life

21.10.1 Contact Fatigue Theory

Rolling fatigue is a material failure caused by the application of repeated stresses to a small volume of material. It is a unique failure type—essentially a process of seeking out the weakest point at which the first failure will occur. A typical spall is shown in Fig. 21.25. On a microscale there will probably be a wide dispersion in material strength or resistance to fatigue because of inhomogeneities in the material. Because bearing materials are complex alloys, they are not homogeneous or equally resistant to failure at all points. Therefore, the fatigue process can be expected to be one in which a group of supposedly identical specimens exhibit wide variations in failure time when stressed in the same way. For this reason it is necessary to treat the fatigue process statistically.

Predicting how long a particular bearing will run under a specific load requires the following two essential pieces of information:

1. An accurate, quantitative estimate of the life dispersion or scatter
2. The life at a given survival rate or reliability level

This translates into an expression for the “load-carrying capacity,” or the ability of the bearing to endure a given load for a stipulated number of stress cycles or revolutions. If a group of supposedly identical bearings is tested at a specific load and speed, the distribution in bearing lives shown in Fig. 21.26 will occur.

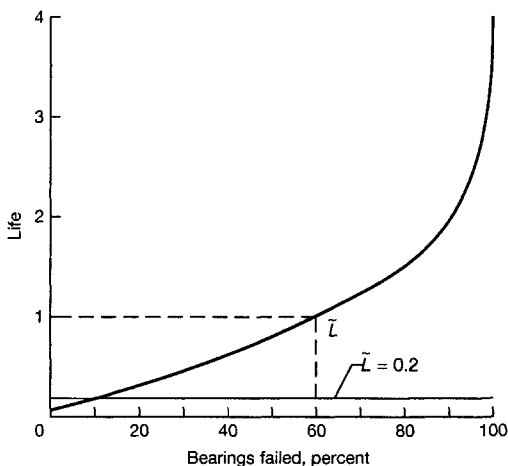


Figure 21.26: Distribution of bearing fatigue failures. [From Hamrock and Anderson (1983)].

21.10.2 Weibull Distribution

Weibull (1949) postulates that the fatigue lives of a homogeneous group of rolling-element bearings are dispersed according to the following relation:

$$\ln \ln \frac{1}{\tilde{S}} = e_1 \ln \frac{\tilde{L}}{\tilde{A}} \quad (21.57)$$

where $\tilde{S}$ is the probability of survival, $\tilde{L}$ is the fatigue life, and e_1 and $\tilde{A}$ are constants. The Weibull distribution results from a statistical theory of strength based on probability theory, where the dependence of strength on volume is explained by the dispersion in material strength. This is the “weakest link” theory.

Consider a volume being stressed that is broken up into m similar volumes



$$\tilde{S}_1 = 1 - \tilde{M}_1 \quad \tilde{S}_2 = 1 - \tilde{M}_2 \quad \tilde{S}_3 = 1 - \tilde{M}_3 \quad \dots \quad \tilde{S}_m = 1 - \tilde{M}_m$$

The $\tilde{M}$ s represent the probability of failure and the $\tilde{S}$ s, the probability of survival. For the entire volume

$$\tilde{S} = \tilde{S}_1 \times \tilde{S}_2 \times \tilde{S}_3 \times \dots \times \tilde{S}_m$$

Then

$$1 - \tilde{M} = (1 - \tilde{M}_1) (1 - \tilde{M}_2) (1 - \tilde{M}_3) \dots (1 - \tilde{M}_m)$$

$$1 - \tilde{M} = \prod_{i=1}^m (1 - \tilde{M}_i)$$

$$\tilde{S} = \prod_{i=1}^m (1 - \tilde{M}_i) \quad (21.58)$$

The probability of a crack starting in the i th volume is

$$\tilde{M}_i = f(x)\tilde{V}_i \quad (21.59)$$

where $f(x)$ is a function of the stress level, the number of stress cycles, and the depth into the material where the maximum stress occurs and $\tilde{V}_i$ is the elementary volume. Therefore, substituting Eq. (21.59) into (21.58) gives

$$\tilde{S} = \prod_{i=1}^m [1 - f(x)\tilde{V}_i]$$

$$\ln \tilde{S} = \sum_{i=1}^m \ln [1 - f(x)\tilde{V}_i]$$

Now if $f(x)\tilde{V}_i \ll 1$, then $\ln [1 - f(x)\tilde{V}_i] = -f(x)\tilde{V}_i$ and

$$\ln \tilde{S} = - \sum_{i=1}^m f(x)\tilde{V}_i \quad (21.60)$$

Let $\tilde{V}_i \rightarrow 0$; then

$$\sum_{i=1}^m f(x)\tilde{V}_i = \int f(x) d\tilde{V} = \bar{f}(x)\tilde{V} \quad (21.61)$$

where $\bar{f}(x)$ is a volume-average value of $f(x)$.

Lundberg and Palmgren (1947) assumed that $f(x)$ could be expressed as a power function of shear stress τ_0 , number of stress cycles $\tilde{J}$, and depth of the maximum shear stress z_0 :

$$\bar{f}(x) = \frac{\tau_0^{c_1} \tilde{J}^{c_2}}{z_0^{c_3}} \quad (21.62)$$

They also chose as the stressed volume

$$\tilde{V} = D_y z_0 \ell_v \quad (21.63)$$

Substituting Eqs. (21.61) to (21.63) into (21.60) gives

$$\ln \tilde{S} = - \frac{\tau_0^{c_1} \tilde{J}^{c_2} D_y \ell_v}{z_0^{c_3-1}}$$

or

$$\ln \frac{1}{\tilde{S}} = \frac{\tau_0^{c_1} \tilde{J}^{c_2} D_y \ell_v}{z_0^{c_3-1}} \quad (21.64)$$

For a specific bearing and load (e.g., stress), τ_0 , D_y , ℓ_v , and z_0 are all constant so that

$$\ln \frac{1}{\tilde{S}} \approx \tilde{J}^{c_2}$$

Designating $\tilde{J}$ as life $\tilde{L}$ in stress cycles gives

$$\ln \frac{1}{\tilde{S}} = \left(\frac{\tilde{L}}{\tilde{A}} \right)^{c_2}$$

or

$$\ln \ln \frac{1}{\tilde{S}} = c_2 \ln \frac{\tilde{L}}{\tilde{A}} \quad (21.65)$$

This is the Weibull distribution, which relates probability of survival and life, it has two principal functions. First, bearing fatigue lives plot as a straight line on Weibull coordinates (log-log versus log) so that the life at any reliability level can be determined. Of most interest are the $\tilde{L}_{10}$ life ($\tilde{S} = 0.9$) and the $\tilde{L}_{50}$ life ($\tilde{S} = 0.5$). Bearing load ratings are based on the $\tilde{L}_{30}$ life. Second, Eq. (21.65) can be used to determine what the $\tilde{L}_{10}$ life must be to obtain a required life at any reliability level. The $\tilde{L}_{10}$ life is calculated, from the load on the bearing and the bearing dynamic capacity or the load rating given in manufacturers' catalogs and engineering journals, by using the equation

$$\tilde{L} = \left(\frac{\tilde{C}}{w_e} \right)^{m_k} \quad (21.66)$$

where

$\tilde{C}$ = basic dynamic capacity or load rating, N

w_e = equivalent bearing load, N

m_k = load-life exponent; 3 for elliptical contacts and 10/3 for rectangular contacts

A typical Weibull plot is shown in Fig. 21.27.

21.10.3 Lundberg-Palmgren Theory

The Lundberg-Palmgren theory, on which bearing ratings are based, is expressed by Eq. (21.64). The exponents in this equation are determined experimentally from the dispersion of bearing lives and the dependence of life on load, geometry, and bearing size. As a standard of reference, all bearing load ratings are expressed in terms of the specific dynamic capacity $\tilde{C}$, which, by definition, is the load that a bearing can carry for 1 million inner-race revolutions with a 90% chance of survival.

Factors on which specific dynamic capacity and bearing life depend are

1. Size of rolling element
2. Number of rolling elements per row

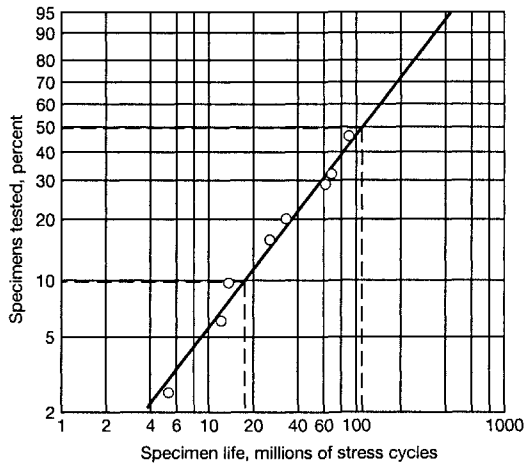


Figure 21.27: Typical Weibull plot of bearing fatigue failures. [From Hamrock and Anderson (1983)].

3. Number of rows of rolling elements
4. Conformity between rolling elements and races
5. Contact angle under load
6. Material properties
7. Lubricant properties
8. Operating temperature
9. Operating speed

Only factors 1 to 5 are incorporated in bearing dynamic capacities developed from the Lundberg-Palmgren theory. The remaining factors must be taken into account in the life adjustment factors discussed in Sec. 21.10.5.

The formulas for specific dynamic capacity as developed by Lundberg and Palmgren (1947, 1952) are as follows:

For radial ball bearings with $d < 25$ mm:

$$\bar{C} = f_c (i \cos \beta)^{0.7} n^{2/3} \left(\frac{d}{0.0254} \right)^{1.8} \quad (21.67)$$

where

d = diameter of rolling element, m

i = number of rows of rolling elements

n = number of rolling elements per row

β = contact angle

f_c = coefficient dependent on material and bearing type

For radial ball bearings with $d \geq 25$ mm:

$$\bar{C} = f_c (i \cos \beta)^{0.7} n^{2/3} \left(\frac{d}{0.0254} \right)^{1.4} \quad (21.68)$$

For radial roller bearings:

$$\bar{C} = f_c (i \cos \beta)^{0.78} n^{3/4} \left(\frac{d}{0.0254} \right)^{1.07} \left(\frac{\ell_t}{0.0254} \right)^{0.78} \quad (21.69)$$

where ℓ_t is roller length in meters.

For thrust ball bearings with $\beta \neq 90^\circ$:

$$\bar{C} = f_c (i \cos \beta)^{0.7} (\tan \beta) n^{2/3} \left(\frac{d}{0.0254} \right)^{1.8} \quad (21.70)$$

For thrust roller bearings with $\beta \neq 90^\circ$:

$$\bar{C} = f_c (i \cos \beta)^{0.78} (\tan \beta) n^{3/4} \left(\frac{\ell_t}{0.0254} \right)^{0.78} \quad (21.71)$$

For thrust ball bearings with $\beta = 90^\circ$:

$$\bar{C} = f_c i^{0.7} n^{2/3} \left(\frac{d}{0.0254} \right)^{1.8} \quad (21.72)$$

For thrust roller bearings with $\beta = 90^\circ$:

$$\bar{C} = f_c i^{0.78} n^{3/4} \left(\frac{d}{0.0254} \right)^{1.07} \left(\frac{\ell_t}{0.0254} \right)^{0.78} \quad (21.73)$$

For ordinary bearing steels such as SAE 52100 with mineral oil lubrication, f_c can be evaluated by using Tables 21.10 and 21.11, but a more convenient method is to use tabulated values from the most recent Antifriction Bearing Manufacturers Association (AFBMA) documents on dynamic load ratings and life (ISO, 1976). The values of $\bar{C}$ appear in bearing manufacturers' catalogs, together with factors $\bar{X}$ and $\bar{Y}$ used to determine the equivalent load w_e . The equivalent load can be calculated from the equation

$$w_e = \bar{X} w_z + \bar{Y} w_x \quad (21.74)$$

In addition to specific dynamic capacity $\bar{C}$, every bearing has a specific static capacity, usually designated as $\bar{C}_0$. Specific static capacity is defined as the load that under static conditions will result in a permanent deformation of 0.0001 times the rolling-element diameter. For some bearings $\bar{C}_0$ is less than $\bar{C}$; therefore it is important to avoid exposing a bearing to a static load that exceeds $\bar{C}_0$. Values of $\bar{C}_0$, are also given in bearing manufacturers' catalogs.

Table 21.10: Capacity formulas for rectangular and elliptical conjunctions. (units in kilograms and millimeters)

Function	Elliptical contact of ball bearings			Rectangular contact of roller bearings		
$\bar{C}$	$f_c f_a i^{0.7} n^{2/3} d^{1.8}$			$f_c f_a i^{7/9} n^{3/4} d^{29/27} \ell_{t,i}^{7/9}$		
f_c	$g_c f_1 f_2 \left(\frac{d_i}{d_i - d} \right)^{0.41}$			$g_c f_1 f_2$		
g_c	$\left[1 + \left(\frac{\bar{C}_i}{\bar{C}_o} \right)^{10/8} \right]^{-0.8}$			$\left[1 + \left(\frac{\bar{C}_i}{\bar{C}_o} \right)^{9/2} \right]^{-2/9}$		
$\bar{C}_i/\bar{C}_o$	$f_3 \left(\frac{f_i (d_o - d)}{d_o (d_i - d)} \right)^{0.41}$			$f_3 \left(\frac{\ell_{t,i}}{\ell_{t,o}} \right)^{7/9}$		
	Radial	Thrust		Radial	Thrust	
		$\beta \neq 90^\circ$	$\beta = 90^\circ$		$\beta \neq 90^\circ$	$\beta = 90^\circ$
γ_n		$\frac{d \cos \beta}{d_e}$	$\frac{d}{d_e}$		$\frac{d \cos \beta}{d_e}$	$\frac{d}{d_e}$
f_a	$(\cos \beta)^{0.7}$	$(\cos \beta)^{0.7} \tan \beta$	1	$(\cos \beta)^{7/9}$	$(\cos \beta)^{7/9} \tan \beta$	1
f_1	3.7-4.1	6-10		18-25	36-60	
f_2	$\frac{\gamma_n^{0.3} (1 - \gamma_n)^{1.39}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{0.3}$	$\frac{\gamma_n^{2/9} (1 - \gamma_n)^{29/27}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{2/9}$
f_3	1.04 f_4	f_4		1.14 f_4	f_4	
f_4	$\left(\frac{1 - \gamma_n}{1 + \gamma_n} \right)^{1.72}$			$\left(\frac{1 - \gamma_n}{1 + \gamma_n} \right)^{38/37}$		

Table 21.11: Capacity formulas for mixed rectangular and elliptical conjunctions. $\{\bar{C} = \bar{C}_i [1 + (\bar{C}_i / \bar{C}_o)^4]^{1/4}$; units in kilograms and millimeters.}

Function	Radial bearing	Thrust bearing		Radial bearing	Thrust bearing	
		$\beta \neq 90^\circ$	$\beta = 90^\circ$		$\beta \neq 90^\circ$	$\beta = 90^\circ$
		Inner race			Outer race	
γ_n	$\frac{d \cos \beta}{d_e}$	$\frac{d}{d_e}$	$\frac{d \cos \beta}{d_e}$	$\frac{d}{d_e}$		
	Rectangular conjunction (C_i)			Elliptical conjunction (C_o)		
$\bar{C}_i$ or $\bar{C}_o$	$f_1 f_2 f_a i^{7/9} n^{3/4} d^{29/27} \ell_{t,i}^{7/9}$			$f_1 f_2 f_a \left(\frac{2R}{D} \frac{r_o}{r_o - R} \right)^{0.41} i^{0.7} n^{2/3} d^{1.8}$		
f_a	$(\cos \beta)^{7/9}$	$(\cos \beta)^{7/9} \tan \beta$	1	$(\cos \beta)^{0.7}$	$(\cos \beta)^{0.7} \tan \beta$	1
f_1	18-25	36-60		3.5-3.9	6-10	
f_2	$\frac{\gamma_n^{2/9} (1 - \gamma_n)^{29/27}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{3/9}$	$\frac{\gamma_n^{0.3} (1 - \gamma_n)^{1.39}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{0.3}$
	Point contact (C_i)			Line contact (C_o)		
$\bar{C}_i$ or $\bar{C}_o$	$f_1 f_2 f_a \left(\frac{2R}{D} \frac{r_i}{r_i - R} \right)^{0.41} i^{0.7} n^{2/3} d^{1.8}$			$f_1 f_2 f_a i^{7/9} n^{3/4} d^{29/27} \ell_{t,o}^{7/9}$		
f_a	$(\cos \beta)^{0.7}$	$(\cos \beta)^{0.7} \tan \beta$	1	$(\cos \beta)^{7/9}$	$(\cos \beta)^{7/9} \tan \beta$	1
f_1	3.7-4.1	6-10		15-22	36-60	
f_2	$\frac{\gamma_n^{0.3} (1 - \gamma_n)^{1.39}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{0.3}$	$\frac{\gamma_n^{2/9} (1 - \gamma_n)^{29/27}}{(1 + \gamma_n)^{1/3}}$		$\gamma_n^{2/9}$

21.10.4 AFBMA Methods

Shortly after publication of the Lundberg-Palmgren theory the AFBMA began efforts to standardize methods for establishing bearing load ratings and making life predictions. Standardized methods of establishing load ratings for ball bearings (AFBMA, 1960b) and roller bearings (AFBMA, 1960a) were devised, based essentially on the Lundberg-Palmgren theory. These early standards are published in their entirety in Jones (1964). In recent years significant advances have been made in rolling-element bearing material quality and in our understanding of the role of lubrication in bearing life through the development of elastohydrodynamic theory. Therefore, the original AFBMA standards in AFBMA (1960a, b) have been updated with life adjustment factors. These factors have been incorporated into ISO (1976), which is discussed in the following section.

21.10.5 Life Adjustment Factors

A comprehensive study of the factors affecting the fatigue life of bearings, which were not taken into account in the Lundberg-Palmgren theory, is reported in Bamberger (1971). In that reference it is assumed that the various environmental or bearing design factors are multiplicative in their effect on bearing life. The following equation results:

$$L_A = (\bar{D})(\bar{E})(\bar{F}_\ell)(\bar{G})(\bar{H}_m)\bar{L}_{10} \quad (21.75)$$

or

$$L_A = (\bar{D})(\bar{E})(\bar{F}_\ell)(\bar{G})(\bar{H}_m)\left(\frac{\bar{C}}{w_e}\right)^{m_k} \quad (21.76)$$

where

$\bar{D}$ = material factor

$\bar{E}$ = metallurgical processing factor

$\bar{F}_\ell$ = lubrication factor

$\bar{G}$ = speed effect factor

$\bar{H}_m$ = misalignment factor

w_e = bearing equivalent load

m_k = load-life exponent; 3 for ball bearings or 10/3 for roller bearings

Factors $\bar{D}$, $\bar{E}$, and $\bar{F}_\ell$ are reviewed briefly here. Refer to Bamberger (1971) for a complete discussion of all five life adjustment factors.

Materials Factors $\bar{D}$ and $\bar{E}$

For over a century AISI 52100 steel has been the predominant material for rolling-element bearings. In fact, the basic dynamic capacity as defined by AFBMA in 1949 is based on an air-melted 52100 steel, hardened to at least Rockwell C 58. Since that time, as discussed in Sec. 5.5, better control of

Table 21.12: Material factor for through-hardened bearing materials. [From Bamberger (1971); air-melted materials assumed.]

Material	Material factor, $\bar{D}$
52100	2.0
M-1	0.6
M-2	0.6
M-10	2.0
M-50	2.0
T-1	0.6
Halmo	2.0
M-42	0.2
WB 49	0.6
440C	0.6-0.8

air-melting processes and the introduction of vacuum-remelting processes have resulted in more homogeneous steels with fewer impurities. Such steels have extended rolling-element bearing fatigue lives to several times the AFBMA (or catalog) life. Life extensions of three to eight times are not uncommon. Other steel compositions, such as AISI M-1 and AISI M-50, chosen for their higher temperature capabilities and resistance to corrosion, also have shown greater resistance to fatigue pitting when vacuum-melting techniques are employed. Case-hardened materials, such as AISI 4620, AISI 4118, and AISI 8620, used primarily for rolling-element bearings, have the advantage of a tough, ductile steel core with a hard, fatigue-resistant surface.

The recommended material factors $\bar{D}$ for various alloys processed by air melting are shown in Table 21.12. Insufficient definitive life data were found for case-hardened materials to recommend values of $\bar{D}$ for them. Refer to the bearing manufacturer for the choice of a specific case-hardened material.

The metallurgical processing variables considered in the development of the metallurgical processing factor $\bar{E}$ included melting practice (air and vacuum melting) and metalworking (thermomechanical working). Thermomechanical working of M-50 has also been shown to lengthen life, but in a practical sense it is costly and still not fully developed as a processing technique. Bamberger (1971) recommends an $\bar{E}$ of 3 for consumable-electrode-vacuum-melted materials.

The translation of these factors into a standard (ISO, 1976) is discussed later.

Lubrication Factor $\bar{F}_\ell$

Until approximately 1960 the role of the lubricant between surfaces in rolling contact was not fully appreciated. Metal-to-metal contact was presumed to occur in all applications with attendant required boundary lubrication. The development of elastohydrodynamic lubrication theory showed that lubricant films with thicknesses on the order of microinches and tenths of microinches

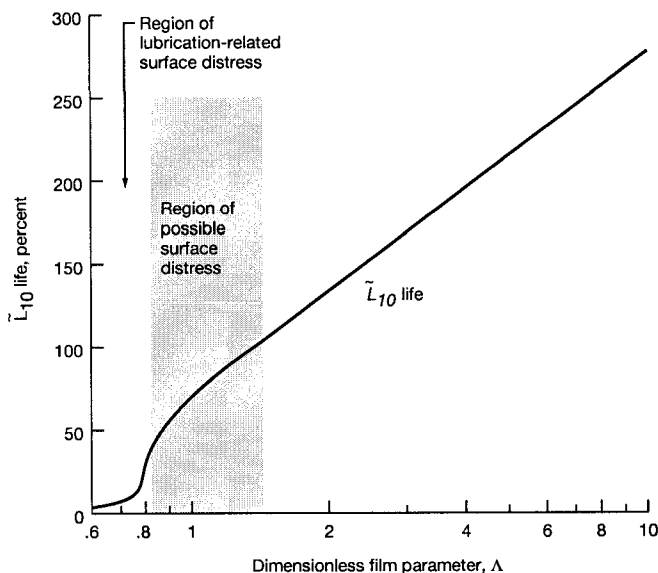


Figure 21.28: Group fatigue life $\tilde{L}_{10}$ as a function of dimensionless film parameter. [From Tallian (1967)].

occur in rolling contact. Since surface finishes are of the same order of magnitude as the lubricant film thicknesses, the significance of rolling-element bearing surface roughnesses to bearing performance became apparent. Tallian (1967) was the first to report on the importance to bearing life of the ratio of elastohydrodynamic lubrication film thickness to surface roughness. Figure 21.28 shows calculated L_{10} life as a function of the dimensionless film parameter Λ , which was introduced in Chapter 3, where

$$\Lambda = \frac{h_{\min}}{\left(R_{q,a}^2 + R_{q,b}^2\right)^{1/2}} \quad (3.22)$$

Figure 21.29, from Bamberger (1971), presents a curve of the recommended $\bar{F}_\ell$ as a function of Λ . A mean of the curves presented in Tallian (1967) for ball bearings and in Skurka (1970) for roller bearings is recommended for use. A formula for calculating the minimum film thickness $h_{\min}$ is given in Eq. (19.18).

The results of Bamberger (1971) have not been fully accepted into the current AFBMA standard represented by ISO (1976). The standard does, however, include the following:

1. Life and dynamic load rating formulas for radial and thrust ball bearings

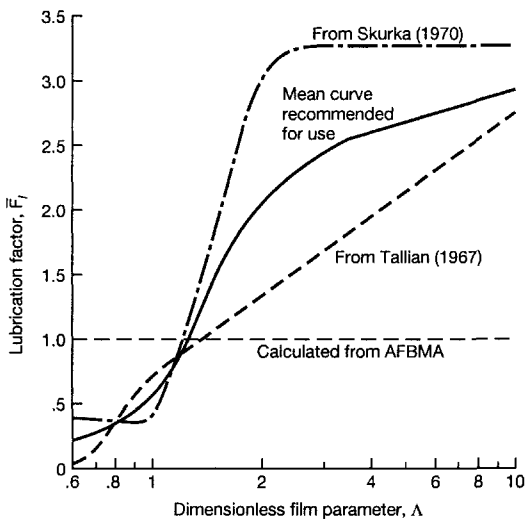


Figure 21.29: Lubrication factor as a function of dimensionless film parameter. [From Bamberger (1971)].

and radial and thrust roller bearings

2. Tables of f_c for all cases
3. Tables of $\tilde{X}$ and $\tilde{Y}$ factors for calculating equivalent loads
4. Load rating formulas for multirow bearings
5. Life correction factors for high-reliability levels $\bar{E}$, materials $\bar{D}$, and lubrication or operating conditions $\bar{F}_\ell$

Procedures for calculating $\bar{D}$ and $\bar{F}_\ell$ are unfortunately at present less than definitive, reflecting the need for additional research, life data, and operating experience.

21.11 Dynamic Analyses and Computer Codes

As has been stated, rolling-element bearing kinematics, stresses, deflections, and life can be analyzed precisely only by using a large-scale computer code. Presented here is a brief discussion of some of the significant analyses that have led to the formulation of modern bearing analytical computer codes. Refer to COSMIC, University of Georgia, Athens, GA 30601, for the public availability of program listings.

21.11.1 Quasi-Static Analyses

Rolling-element bearing analysis began with the work of Jones on ball bearings (Jones, 1959, 1960). Jones did his work before there was a general awareness of

elastohydrodynamic lubrication, and he assumed Coulomb friction in the race contacts. This led to the commonly known “race control” theory, which assumes that pure rolling (except for Heathcote interfacial slip) can occur at one of the ball-race contacts. All the spinning required for dynamic equilibrium of the balls would then take place at the other, or “noncontrolling,” race contact. Jones’ analysis proved to be quite effective for predicting fatigue life but less useful for predicting cage slip, which usually occurs at high speeds and light loads. Harris (1971b) extended Jones’ analysis, retaining the assumption of Coulomb friction but allowing a frictional resistance to gyroscopic moments at the noncontrolling as well as the controlling race contact. Harris’ analysis (1971b) is adequate for predicting bearing performance under conditions of dry-film lubrication or whenever there is a complete absence of any elastohydrodynamic film.

Harris (1971a) first incorporated elastohydrodynamic relationships into a ball bearing analysis. A revised version of Harris’ computer program, called SHABERTH, was developed that incorporated actual traction data from a disk machine. The program SHABERTH has been expanded until today it encompasses ball, cylindrical roller, and tapered-roller bearings.

Harris (1966) first introduced elastohydrodynamics into a cylindrical roller bearing analysis. His initial analysis has been augmented with more precise viscosity-pressure and temperature relationships and traction data for the lubricant. This augmented analysis has evolved into the program CYBEAN and more recently has been incorporated into SHABERTH. Parallel efforts by Harris’ associates have resulted in SPHERBEAN, a program that can be used to predict the performance of spherical roller bearings. These analyses can range from relatively simple force balance and life analyses through a complete thermal analysis of a shaft bearing system in several steps of varying complexity.

21.11.2 Dynamic Analyses

The work of Jones and Harris is categorized as quasi-static because it applies only when steady-state conditions prevail. Under highly transient conditions, such as accelerations or decelerations, only a true dynamic analysis will suffice. Walters (1971) made the first attempt at a dynamic analysis to explain cage dynamics in gyro-spin-axis ball bearings. Gupta (1975) solved the generalized differential equations of ball motion in an angular-contact ball bearing. Gupta continued his dynamic analyses for both cylindrical roller bearings (1979a, b) and ball bearings (1979c, d). Gupta’s work is available in the programs DREB and RAPIDREB. Thus, the development of codes with real-time dynamic simulations that integrate the classical differential equations of motion for each bearing element has followed in the wake of the quasi-static codes. Contributors to dynamic code development are Walters (1971), Gupta (1979a-d, 1984), Gupta et al. (1985, 1986), Gupta and Tallian (1990), and Meeks and Ng (1985a, b). They have produced powerful computational tools that are useful in analyzing transient dynamics.

21.12 Ioannides-Harris Theory

Ioannides and Harris (1985) extended the Lundberg-Palmgren theory on the basis of an elemental calculation of the risk of fatigue and the use of a material fatigue limit, akin to an endurance limit below which fatigue will not occur. They obtained the survival probability $\Delta\tilde{S}$ to N_c cycles of a volume element ΔV of material under known conditions of stress to be

$$-\ln \Delta\tilde{S} \sim N_c^{c_1} (\tau - \tau_u)^{c_2} (z')^{-c_3} \Delta V \quad (21.77)$$

This resembles the classic Lundberg-Palmgren method covered earlier with exponents c_1 , c_2 , and c_3 , but Eq. (21.77) incorporates a local Weibull endurance strength or threshold stress τ_u , and a stress-weighted mean depth z' to replace the maximum shear stress depth z_0 . Equation (21.77) is not built on a detailed model of damage (e.g., the crack propagation and spall formation mechanisms treated by Tallian et al., 1978). Instead, by considering the stress distribution throughout the whole material, the need for an asperity model and the problem of distinguishing competing failure modes originating in the surface and sub-surface are avoided. The characteristic stress σ may be selected from among several criteria, such as a single shear stress component, maximum shear stress, or von Mises.

Integrating Eq. (21.77) over the region in which the shear stress τ exceeds the threshold stress and setting $\tilde{S} = 0.9$ as appropriate to $\tilde{L}_{10}$ life, the following expression for life is obtained:

$$\tilde{L}_{10} = A_{av} \left(\frac{(\tau - \tau_u)^{c_2}}{(z')^{c_3}} dV \right)^{-1/c_1} \quad (21.78)$$

The coefficient A_{av} contains information on average failure risk that is only partially accessible, such as local material property variations or lubricant effects. Fig. 21.30 gives a load-life comparison for the Lundberg-Palmgren and Ioannides-Harris theories. The impact of the different theories is easily seen from this figure.

21.13 Applications

In this section two applications of the film thickness equations developed throughout this chapter are presented to illustrate how the fluid film lubrication conditions in machine elements can be analyzed. Specifically, a typical roller and a typical ball bearing problem are considered.

21.13.1 Cylindrical Roller Bearing Example

The equations for elastohydrodynamic film thickness developed in Chapter 19 relate primarily to elliptical conjunctions, but they are sufficiently general to allow them to be used with adequate accuracy in line-contact problems, as

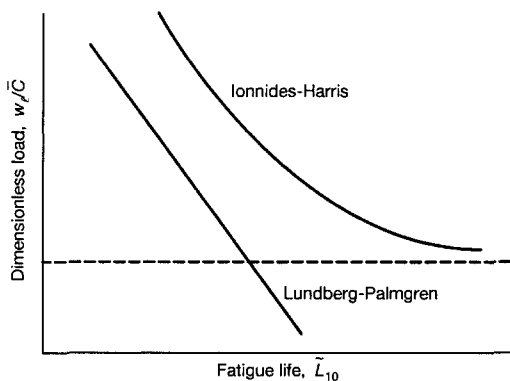


Figure 21.30: Dimensionless load-life comparison for Lundberg-Palmgren and Ioannides-Harris theories. [From Ioannides and Harris (1985)].

would be found in a cylindrical roller bearing. Therefore, the minimum elastohydrodynamic film thicknesses are calculated on the inner and outer races of a cylindrical roller bearing with the following dimensions—for both the elliptical conjunction of Chapter 19 and the rectangular conjunction of Chapter 18:

Inner-race diameter d_i , mm	64
Outer-race diameter d_o , mm	96
Diameter of cylindrical rollers d , mm	16
Axial length of cylindrical rollers ℓ , mm	16
Number of rollers in complete bearing n	9

A bearing of this kind might well experience the following operating conditions:

Radial load w_z , N	10,800
Inner-race angular velocity ω_i , rad/s	524
Outer-race angular velocity ω_o , rad/s	0
Absolute viscosity at $p = 0$ and bearing effective operating temperature, η_0 , N·s/m ²	0.01
Viscosity-pressure coefficient ξ , m ² /N	2.2×10^{-8}
Modulus of elasticity for both rollers and races E , N/m ²	2.075×10^{11}
Poisson's ratio, ν	0.3

Since the diametral clearance c_d is zero, from Eq. (21.47) the most heavily loaded roller can be expressed as

$$(w_z)_{\max} = \frac{4w_z}{n} = \frac{4(10,800 \text{ N})}{9} = 4800 \text{ N} \quad (21.47)$$

Therefore, the radial load per unit length on the most heavily loaded roller is

$$(w'_z)_{\max} = \frac{4800 \text{ N}}{0.016 \text{ m}} = 0.3 \text{ MN/m}$$

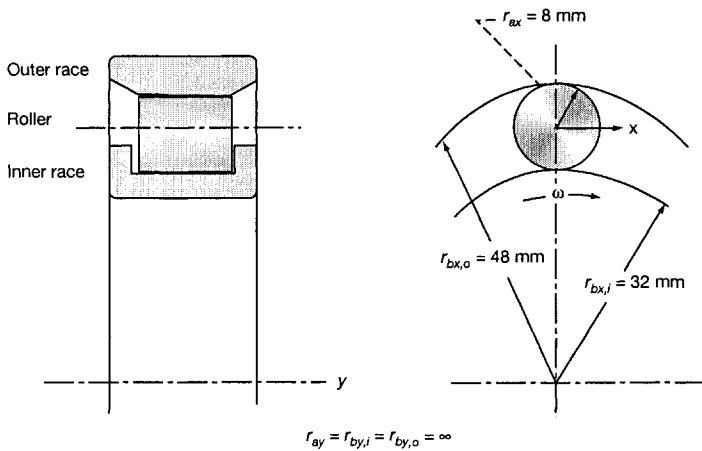


Figure 21.31: Roller bearing example. [From Hamrock and Anderson (1983)].

From Fig. 21.31 the radii of curvature are

$$\begin{aligned} r_{ax} &= 0.008 \text{ m} & r_{ay} &= \infty \\ r_{bx,i} &= 0.032 \text{ m} & r_{by,i} &= \infty \\ r_{bx,o} &= -0.048 \text{ m} & r_{by,o} &= \infty \end{aligned}$$

Then

$$\frac{1}{R_{x,i}} = \frac{1}{0.008} + \frac{1}{0.032} = \frac{5}{0.032} \quad (17.4)$$

giving $R_{x,i} = 0.0064 \text{ m}$,

$$\frac{1}{R_{x,o}} = \frac{1}{0.008} - \frac{1}{0.048} = \frac{5}{0.048} \quad (17.4)$$

giving $R_{x,o} = 0.0096 \text{ m}$ and

$$\frac{1}{R_{y,i}} = \frac{1}{R_{y,o}} = \frac{1}{\infty} + \frac{1}{\infty} = 0 \quad (17.5)$$

giving $R_{y,i} = R_{y,o} = \infty$

From the input information the effective modulus of elasticity can be written as

$$E' = \frac{2}{(1 - \nu_a^2)/E_a + (1 - \nu_b^2)/E_b} = 2.28 \times 10^{11} \text{ N/m}^2 \quad (17.17)$$

For pure rolling the surface velocity $\tilde{u}$ relative to the lubricated conjunctions for a cylindrical roller is

$$\tilde{u} = \frac{|\omega_i - \omega_o| (d_e^2 - d^2)}{4d_e} \quad (21.29)$$

where d_e is the pitch diameter and d is the roller diameter.

$$d_e = \frac{d_o + d_i}{2} = \frac{0.096 + 0.064}{2} = 0.08 \text{ m} \quad (21.1)$$

Hence,

$$\tilde{u} = \frac{0.08^2 - 0.16^2}{4 \times 0.08} |524 - 0| = 10.061 \text{ m/s} \quad (21.28)$$

The dimensionless speed, materials, and load parameters for the inner- and outer-race conjunctions thus become, when the velocity is only in the rolling direction ($\tilde{v} = 0$),

$$U_i = \frac{\eta_0 \tilde{u}}{E' R_{x,i}} = \frac{(0.01)(10.061)}{(2.28 \times 10^{11})(0.0064)} = 6.895 \times 10^{-11} \quad (19.13)$$

$$G_i = \xi E' = 5016 \quad (19.15)$$

$$W_i = \frac{w_z}{E' (R_{x,i})^2} = \frac{4800}{(2.28 \times 10^{11})(0.0064)^2} = 5.140 \times 10^{-4} \quad (19.12)$$

$$U_o = \frac{\eta_0 \tilde{u}}{E' R_{x,o}} = \frac{(0.01)(10.061)}{(2.28 \times 10^{11})(0.0096)} = 4.597 \times 10^{-11} \quad (19.13)$$

$$G_o = \xi E' = 5016 \quad (19.15)$$

$$W_o = \frac{w_z}{E' (R_{x,o})^2} = \frac{4800}{(2.28 \times 10^{11})(0.0096)^2} = 2.284 \times 10^{-4} \quad (19.12)$$

The appropriate elastohydrodynamic film thickness equation for a fully flooded elliptical conjunction is developed in Chapter 19 and recorded as Eq. (19.18):

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_x} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \quad (19.18)$$

In the case of a roller bearing $k = \infty$, and this equation reduces to

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_x} = 3.63 U^{0.68} G^{0.49} W^{-0.073}$$

The dimensionless film thickness for the roller-inner-race conjunction is

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_{x,i}} = (3.63) (1.231 \times 10^{-7}) (65.04)(1.738) = 50.5 \times 10^{-6}$$

and hence

$$\tilde{h}_{\min} = (0.0064) (50.5 \times 10^{-6}) = 0.32 \mu\text{m}$$

The dimensionless film thickness for the roller-outer-race conjunction is

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_{x,o}} = (3.63) (9.343 \times 10^{-8}) (65.04)(1.844) = 40.7 \times 10^{-6}$$

and hence

$$\tilde{h}_{\min} = (0.0096) (40.7 \times 10^{-6}) = 0.39 \mu\text{m}$$

It is clear from these calculations that the smaller minimum film thickness in the bearing occurs at the roller-inner-race conjunction, where the geometrical conformity is less favorable. It was found that if the ratio of minimum film thickness to composite surface roughness is greater than 3, an adequate elastohydrodynamic film is maintained. This implies that a composite surface roughness of less than $0.1 \mu\text{m}$ is needed to ensure that an elastohydrodynamic film is maintained.

Now using, instead of the elliptical conjunction results of Chapter 19, the rectangular conjunction results of Chapter 18 gives

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_x} = 1.714 (W')^{-0.128} U^{0.694} G^{0.568} \quad (18.72)$$

where

$$W' = \frac{w'_z}{E' R_x}$$

For the roller-inner-race conjunction

$$W' = \frac{w'_z}{E' R_x} = \frac{0.3 \times 10^6}{(2.28 \times 10^{11}) (0.0064)} = 2.056 \times 10^{-4}$$

$$\begin{aligned} \tilde{H}_{e,\min} &= (1.714) (2.056 \times 10^{-4})^{-0.128} (6.895 \times 10^{-11})^{0.694} (5016)^{0.568} \\ &= 56.98 \times 10^{-6} \end{aligned}$$

$$h_{\min} = \tilde{H}_{e,\min} R_{x,i} = (56.98 \times 10^{-6}) (0.064) = 0.365 \mu\text{m}$$

For the roller-outer-race conjunction

$$W' = \frac{w'_z}{E' R_x} = \frac{0.3 \times 10^6}{(2.28 \times 10^{11}) (0.0096)} = 1.371 \times 10^{-4}$$

$$\begin{aligned} \tilde{H}_{e,\min} &= (1.714) (1.371 \times 10^{-4})^{-0.128} (4.597 \times 10^{-11})^{0.694} (5016)^{0.568} \\ &= 45.29 \times 10^{-6} \end{aligned}$$

$$h_{\min} = (45.29 \times 10^{-6}) (0.0096) = 0.4348 \mu\text{m}$$

The film thickness obtained from Chapter 18, the rectangular conjunction results, produces approximately a 12% increase from the elliptical conjunction results obtained from Chapter 19. Recall from Chapter 19 that the theoretical results for the elliptical conjunction were 23% smaller than the Koye and Winer (1977) experimental results. The Koye and Winer (1980) experimental results appear to be in better agreement with the rectangular conjunction results presented in Chapter 18. Therefore, it is recommended that the results of Chapter 18 [Eq. (18.72)] be used to evaluate the minimum film thickness for a roller bearing.

21.13.2 Radial Ball Bearing Example

Consider a single-row, radial, deep-groove ball bearing with the following dimensions:

Inner-race diameter d_i , m	0.052291
Outer-race diameter d_o , m	0.077706
Ball diameter d , m	0.012700
Number of balls in complete bearing n	9
Inner-groove radius r_i , m	0.006604
Outer-groove radius r_o , m	0.006604
Contact angle β , deg	0
rms surface finish of balls $R_{q,b}$, μm	0.0625
rms surface finish of races $R_{q,a}$, μm	0.175

A bearing of this kind might well experience the following operating conditions:

Radial load w_z , N	8900
Inner-race angular velocity ω_i , rad/s	400
Outer-race angular velocity ω_o , rad/s	0
Absolute viscosity at $p = 0$ and bearing effective operating temperature η_0 , N·s/m ²	0.04
Viscosity-pressure coefficient ξ , m ² /N	2.3×10^{-8}
Modulus of elasticity for both balls and races E , N/m ²	2×10^{11}
Poisson's ratio for both balls and races ν	0.3

The essential features of the geometry of the inner and outer conjunctions (Figs. 21.1 and 21.6) can be ascertained as follows:

Pitch diameter:

$$d_e = \frac{1}{2} (d_o + d_i) = 0.065 \text{ m} \quad (21.1)$$

Diametral clearance:

$$c_d = d_o - d_i - 2d = 1.5 \times 10^{-5} \text{ m} \quad (21.2)$$

Race conformity:

$$R_{r,i} = R_{r,o} = \frac{r}{d} = 0.52 \quad (21.3)$$

Equivalent radii:

$$R_{x,i} = \frac{d(d_e - d \cos \beta)}{2d_e} = 0.00511 \text{ m} \quad (21.18)$$

$$R_{x,o} = \frac{d(d_e + d \cos \beta)}{2d_e} = 0.00759 \text{ m} \quad (21.20)$$

$$R_{y,i} = \frac{R_{r,i}d}{2R_{r,i} - 1} = 0.165 \text{ m} \quad (21.19)$$

$$R_{y,o} = \frac{R_{r,o}d}{2R_{r,o} - 1} = 0.165 \text{ m} \quad (21.21)$$

The curvature sum of inner-ring and ball contact

$$\frac{1}{R_i} = \frac{1}{R_{x,i}} + \frac{1}{R_{y,i}} \text{ m}^{-1} = 201.76 \text{ m}^{-1} \quad (17.2)$$

gives $R_i = 4.956 \times 10^{-3} \text{ m}$, and the curvature sum of outer-ring and ball contact

$$\frac{1}{R_o} = \frac{1}{R_{x,o}} + \frac{1}{r_{y,o}} \text{ m}^{-1} = 137.81 \text{ m}^{-1} \quad (17.2)$$

gives $R_o = 7.256 \times 10^{-3} \text{ m}$. Also, $\alpha_{r,i} = R_{y,i}/R_{x,i} = 32.35$ and $\alpha_{r,o} = R_{y,o}/R_{x,o} = 21.74$.

The nature of the Hertzian contact conditions can now be assessed:

Ellipticity parameters:

$$\begin{aligned} \bar{k}_i &= \alpha_{r,i}^{2/\pi} = 9.15 \\ \bar{k}_o &= \alpha_{r,o}^{2/\pi} = 7.09 \\ q_a &= \frac{\pi}{2} - 1 \end{aligned} \quad (17.29)$$

Elliptic integrals:

$$\begin{aligned} \bar{\mathcal{E}}_i &= 1 + \frac{q_a}{\alpha_{r,i}} = 1.0176 \\ \bar{\mathcal{E}}_o &= 1 + \frac{q_a}{\alpha_{r,o}} = 1.0263 \end{aligned} \quad (17.30)$$

$$\begin{aligned} \bar{\mathcal{F}}_i &= \frac{\pi}{2} + q_a \ln \alpha_{r,i} = 3.555 \\ \bar{\mathcal{F}}_o &= \frac{\pi}{2} + q_a \ln \alpha_{r,o} = 3.3284 \end{aligned} \quad (17.32)$$

The effective elastic modulus E' is given by

$$E' = \frac{2}{(1 - \nu_a^2)/E_a + (1 - \nu_b^2)/E_b} = 2.198 \times 10^{11} \text{ N/m}^2$$

To determine the load carried by the most heavily loaded ball in the bearing, it is necessary to adopt an iterative procedure based on the calculation of local static compression and the analysis presented in Sec. 21.7.2. Stribeck (1901) found that the value of Z_w was about 4.37 in the expression

$$(w_z)_{\max} = \frac{Z_w w_z}{n} \quad (21.45)$$

where

$$\begin{aligned} (w_z)_{\max} &= \text{load on most heavily loaded ball} \\ w_z &= \text{radial load on bearing} \\ n &= \text{number of balls} \end{aligned}$$

However, it is customary to adopt a value of $Z_w = 5$ in simple calculations to produce a conservative design, and this value will be used to begin the iterative procedure.

Stage 1. Assume $Z_w = 5$. Then,

$$(w_z)_{\max} = \frac{5w_z}{9} = \frac{5}{9} \times 8900 = 4944 \text{ N} \quad (21.45)$$

The maximum local elastic compression is

$$\begin{aligned} \delta_i &= \mathcal{F}_i \left[\frac{9}{2\mathcal{E}_i R_i} \left(\frac{(w_z)_{\max}}{\pi k_i E'} \right)^2 \right]^{1/3} = 2.906 \times 10^{-5} \text{ m} \\ \delta_o &= \mathcal{F}_o \left[\frac{9}{2\mathcal{E}_o R_o} \left(\frac{(w_z)_{\max}}{\pi k_o E'} \right)^2 \right]^{1/3} = 2.832 \times 10^{-5} \text{ m} \end{aligned} \quad (17.16)$$

The sum of the local compressions on the inner and outer races is

$$\delta = \delta_i + \delta_o = 5.738 \times 10^{-5} \text{ m}$$

A better value for Z_w can now be obtained from

$$Z_w = \frac{\pi (1 - c_d/2\delta)^{3/2}}{2.491 \left\{ \left[1 + \left(\frac{1 - c_d/2\delta}{1.23} \right)^2 \right]^{1/2} - 1 \right\}}$$

since $c_d/2\delta = (1.5 \times 10^{-5})/(5.779 \times 10^{-5}) = 0.1298$. Thus, from Eq. (21.46)

$$Z_w = 4.551$$

Stage 2.

$$Z_w = 4.551$$

$$(w_z)_{\max} = \frac{4.551 \times 8900}{9} = 4500 \text{ m}$$

$$\delta_i = 2.725 \times 10^{-5} \text{ m}$$

$$\delta_o = 2.702 \times 10^{-5} \text{ m}$$

$$\delta = 5.427 \times 10^{-5} \text{ m}$$

Thus

$$Z_w = 4.565$$

Stage 3.

$$Z_w = 4.565$$

$$(w_z)_{\max} = \frac{4.565 \times 8900}{9} = 4514 \text{ N}$$

$$\delta_i = 2.731 \times 10^{-5} \text{ m}$$

$$\delta_o = 2.708 \times 10^{-5} \text{ m}$$

$$\delta = 5.439 \times 10^{-5} \text{ m}$$

and hence

$$Z_w = 4.564$$

This value is close to the previous value from stage 2 of 4.565, and a further iteration confirms its accuracy.

Stage 4.

$$Z_w = 4.564$$

$$(w_z)_{\max} = \frac{4.564 \times 8900}{9} = 4513 \text{ N}$$

$$\delta_i = 2.731 \times 10^{-5} \text{ m}$$

$$\delta_o = 2.707 \times 10^{-5} \text{ m}$$

$$\delta = 5.438 \times 10^{-5} \text{ m}$$

$$\frac{c_d}{2\delta} = 0.1379$$

and hence

$$Z_w = 4.564$$

The load on the most heavily loaded ball is thus 4513 N.

The elastohydrodynamic minimum film thickness is calculated as follows: For pure rolling,

$$\tilde{u} = \frac{|\omega_o - \omega_i| (d_e^2 - d^2)}{4d_e} = 6.252 \text{ m/s} \quad (21.29)$$

The dimensionless speed, materials, and load parameters for the inner- and outer-race conjunctions thus become, when the velocity is only in the rolling direction ($\tilde{v} = 0$),

$$U_i = \frac{\eta_0 \tilde{u}}{E' R_{x,i}} = \frac{0.04 \times 6.252}{(2.198 \times 10^{11}) (5.11 \times 10^{-3})} = 2.227 \times 10^{-10} \quad (19.13)$$

$$G_i = \xi E' = (2.3 \times 10^{-8}) (2.198 \times 10^{11}) = 5055 \quad (19.15)$$

$$W_i = \frac{w_z}{E' (r_{x,i})^2} = \frac{4513}{(2.198 \times 10^{11}) (5.11^2 \times 10^{-6})} = 7.863 \times 10^{-4} \quad (19.12)$$

$$U_o = \frac{\eta_0 \tilde{u}}{E' R_{x,o}} = \frac{0.04 \times 6.252}{(2.198 \times 10^{11}) (7.59 \times 10^{-3})} = 1.499 \times 10^{-10} \quad (19.13)$$

$$G_o = \xi E' = (2.3 \times 10^{-8}) (2.198 \times 10^{11}) = 5055 \quad (19.15)$$

$$W_o = \frac{w_z}{E' (r_{x,o})^2} = \frac{4513}{(2.198 \times 10^{11}) (7.59^2 \times 10^{-6})} = 3.564 \times 10^{-4} \quad (19.12)$$

The dimensionless minimum elastohydrodynamic film thickness in a fully flooded elliptical conjunction is again obtained from Eq. (19.18).

$$\tilde{H}_{e,\min} = \frac{\tilde{h}_{\min}}{R_x} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \quad (19.18)$$

Ball-inner-race conjunction:

$$\left(\tilde{H}_{e,\min} \right)_i = (3.63) (2.732 \times 10^{-7}) (65.29) (1.685) (0.9983) = 1.09 \times 10^{-4} \quad (19.18)$$

Thus,

$$\tilde{h}_{\min} = (1.09 \times 10^{-4}) R_{x,i} = 0.557 \mu\text{m}$$

The dimensionless film parameter Λ discussed in Sec. 3.9 was found to play a significant role in determining the fatigue life of rolling-element bearings. In this case

$$\Lambda = \frac{\left(\tilde{h}_{rmin} \right)_i}{\left(R_{q,a}^2 + R_{q,b}^2 \right)^{1/2}} = \frac{0.557 \times 10^{-6}}{\left[(0.175)^2 + (0.0625)^2 \right]^{1/2} \times 10^{-6}} = 3.00 \quad (3.22)$$

Ball-outer-race conjunction:

$$\begin{aligned} \left(\tilde{H}_{\min} \right)_o &= \frac{\left(\tilde{h}_{\min} \right)_o}{R_{x,o}} = 3.63 U_o^{0.68} G_o^{0.49} W_o^{-0.073} (1 - e^{-0.68k_o}) \\ &= (3.63) (2.087 \times 10^{-7}) (65.29) (1.785) (0.9919) \\ &= 0.876 \times 10^{-4} \end{aligned}$$

Thus,

$$(h_{\min})_o = 0.876 \times 10^{-4} R_{x,o} = 0.665 \mu\text{m}$$

In this case the dimensionless film parameter is given by

$$\Lambda = \frac{0.665 \times 10^{-6}}{\left[(0.175)^2 + (0.0625)^2 \right]^{1/2} \times 10^{-6}} = 3.58 \quad (3.22)$$

It is evident that the smaller minimum film thickness occurs between the most heavily loaded ball and the inner race. However, in this case the minimum elastohydrodynamic film thickness is about three times the composite surface roughness, and the bearing lubrication can be deemed to be entirely satisfactory. Indeed, it is clear from Fig. 21.29 that little improvement in the lubrication factor $\bar{F}_\ell$ and thus in the fatigue life of the bearing could be achieved by further improving the minimum film thickness and hence the dimensionless film parameter.

21.14 Closure

Rolling-element bearings are precise, yet simple machine elements of great utility. This chapter drew together the current understanding of rolling-element bearings and attempted to present it in a concise manner. The material presented the operation of rolling-element bearings; the detailed precise calculations of bearing performance were only summarized and appropriate references given. The history of rolling-element bearings was briefly reviewed, and subsequent sections described the types of rolling-element bearing, their geometry, and their kinematics. Having defined conditions of a ball bearing under unloaded and unlubricated conditions, the chapter then focused on static loading of rolling-element bearings. Most rolling-element bearing applications involve steady-state rotation of either the inner or outer race or both; however, the rotational speeds are usually not so great as to cause ball or roller centrifugal forces or gyroscopic moments of significant magnitudes. Thus, these were neglected. Radial, thrust, and preloaded bearings that are statically loaded were considered. Rolling friction and friction in bearings concluded the second major thrust of the chapter, which was on loaded but unlubricated rolling element bearings.

The last major thrust of the chapter dealt with loaded and lubricated rolling-element bearings. Topics covered were lubrication systems, fatigue life, dynamic analyses, and computer codes. The chapter concluded by applying the knowledge of this and previous chapters to roller and ball bearing applications. The use of the elastohydrodynamic lubrication film thickness developed in Chapters 18 and 19 was integrated with the rolling-element bearing ideas developed in this chapter. It was found that the most critical conjunctions of both ball and roller bearings occurred between the rolling elements and the inner races.

21.15 Problems

21.1 Outline the considerations that govern the following aspects of the performance of rolling-element bearings:

- (a) Load capacity

- (b) Rotational speed
- (c) Life
- (d) Lubrication requirements

21.2 The geometry of the rolling surfaces in rolling-element bearings is determined from the load-carrying capacity of the elements and the type of motion. The load in ball bearings is carried by the balls. Depending on the geometry of the races, either radial, axial, or moment loads can be transmitted through a ball bearing.

Describe which ball bearing type should be used for

- (a) Pure radial load
- (b) Pure axial load
- (c) Bending moment load
- (d) High radial stiffness
- (e) High axial stiffness

21.3 A single-row, deep-groove ball bearing, shown in Fig. 21.1, has a pitch diameter of 100 mm, an inner-race diameter of 80 mm, and a ball diameter of 19.9 mm. The outer-race radius is 10.5 mm, and the inner-race radius is 10.2 mm. The shoulder height angle is 30° . Determine

- (a) Diametral clearance
- (b) Inner- and outer-race conformities as well as total conformity
- (c) Free contact angle and endplay
- (d) Shoulder heights for the inner and outer races
- (e) Ellipticity parameter and the curvature sum and difference of the contacts

21.4 As shown in Fig. 21.12 the motion of the balls can be guided by the inner-race contact or by the outer-race contact or by both. This event is dependent on both the surface roughness and the surface geometry of angular-contact bearings. At extremely high speeds the centrifugal forces acting on the balls change both the contact angle of the outer race and the friction forces between the ball and the outer race. The outer-race contact angle is 20° and the inner-race contact angle is 31° . The ball diameter is 14 mm and the pitch diameter is 134 mm.

Find the angular speed of the balls and the separator in an angular contact ball bearing at an inner race speed of 10,000 rpm when

- (a) The balls are outer-race guided
- (b) They are inner-race guided

- 21.5 A lightly loaded lubricated roller rolls on a flat surface. The minimum oil film thickness is directly proportional to the roller radius. The cage pocket for a cylindrical roller bearing can be made with straight sides or with sides having a radius 5% larger than the roller radius. Find how much thicker the oil film between the roller and the pockets will be with a radius than with the straight pockets.
- 21.6 A single-row, angular contact ball bearing is loaded with a static load having an axial component of 1000 N and a radial component of 500 N. The static load rating is 1500 N. Find the contact angle between 20 and 40° that produces the largest safety factor.
- 21.7 A deep-groove ball bearing has 12 balls radially loaded with 12,000 N. The groove and the ball radii are matched to give an ellipticity ratio of 10. The lubricant viscosity is 0.075 N·s/m², and its pressure-viscosity coefficient is 1.6×10^{-8} m²/N. The rotational speed is 1500 rpm, the mean surface velocity is 3 m/s, and the effective radius in the *x*-direction is 8 mm. The modulus of elasticity for the ball and races is 207 GPa and Poisson's ratio is 0.3. Find the minimum oil film thickness in the bearing.
- 21.8 A ball bearing is having problems with an oil film that is too thin. The geometries are such that the ellipticity parameter is 2. By decreasing the race radius of curvature in the axial direction, the balls fit more tightly with the races and thus the ellipticity parameter increases from 2 to 10. Find how much thicker the oil film becomes in changing the ellipticity parameter from 2 to 10. Assume that all the other operating parameters remain constant.
- 21.9 A deep-groove 6304 ball bearing has a load rating of 15,900 N. It is radially loaded with 4000 N and axially loaded with 2000 N. The rotational speed is 3000 rpm. Find how many hours the bearing will survive with 90% probability.
- 21.10 A machine manufacturer has dimensioned the roller bearings in a paper-making machine containing 400 bearings such that the $\tilde{L}_{10}$ for each bearing is 40 years. Use Fig. 21.27 to calculate the $\tilde{L}_{50}$ life in years. Also, determine the mean time between failures in the steady state for an old paper-making machine where bearings are replaced when they fail.

References

- AFBMA (1960a): Method of Evaluating Load Ratings for Roller Bearings. AFBMA Standard System No. 11. Anti-Friction Bearings Manufacturers Association, Inc., New York.
- AFBMA (1960b): Method of Evaluating Load Ratings for Ball Bearings. AFBMA Standard System No. 9, Revision No. 4. Anti-Friction Bearing Manufacturers Association, Inc., New York.

- Anderson, W. J. (1970): Elastohydrodynamic Lubrication Theory as a Design Parameter for Rolling Element Bearings. *ASME, Pap. 70-DE-19*.
- Anderson, W. J. (1979): Practical Impact of Elastohydrodynamic Lubrication. in *Elastohydrodynamics and Related Topics*. Proceedings of Fifth Leeds-Lyon Symposium in Tribology. D. Dowson et al. (eds.). Mechanical Engineering Publications, Bury St. Edmunds, Suffolk, England, pp. 217-226.
- Anderson, W. J., Macks. E. F., and Nemeth. Z. N. (1954): Comparison of Performance of Experimental and Conventional Cage Designs and Materials for 75-Millimeter-Bore Cylindrical Roller Bearings at High Speeds. *NACA Rep. 1177*.
- Bamberger, E. N. (1971): *Life Adjustment Factors for Ball and Roller Bearings-An Engineering Design Guide*. American Society for Mechanical Engineers, New York.
- Bisson, E. E., and Anderson, W. J. (1964): Advanced Bearing Technology. *NASA Spec. Publ. 38*.
- Brown, P. F. (1970): Bearings and Dampers for Advanced Jet Engines. *SAE Pap. 700318*.
- Brown, P. F., et al. (1977): Development of Mainshaft High Speed Cylindrical Roller Bearings for Gas Turbine Engines. PWA-FR-8615, Pratt & Whitney Group, West Palm Beach, Florida. (Avail. NTIS, AD-A052351.)
- Gupta, P. K. (1975): Transient Ball Motion and Skid in Ball Bearings. *J. Lubr. Technol.*, vol. 97, no. 2, pp. 261-269.
- Gupta, P. K. (1979a): Dynamics of Rolling Element Bearings-Part 1. Cylindrical Roller Bearing Analysis. *J. Lubr. Technol.*, vol. 101, no. 3, pp. 293-304.
- Gupta. P. K. (1979b): Dynamics of Rolling Element Bearings-Part II. Cylindrical Roller Bearing Results. *J. Lubr. Technol.*, vol. 101, no. 3. pp. 305-311.
- Gupta, P. K. (1979c): Dynamics of Rolling Element Bearings-Part III, Ball Bearing Analysis. *J. Lubr. Technol.*, vol. 101, no. 3, pp. 312-318.
- Gupta, P. K. (1979d): Dynamics of Rolling Element Bearings-Part IV, Ball Bearing Results. *J. Lubr. Technol.*, vol. 101, no. 3, pp. 319-326.
- Gupta, P. K. (1984): *Advanced Dynamics of Rolling Elements*, Springer-Verlag, New York.
- Gupta, P. K., and Tallian, T. E. (1990): Rolling Bearing Life Prediction-Correction for Materials and Operating Conditions-Part III: Implementation in Bearing Dynamic Computer Codes. *J. Tribol.*, vol. 112, no. 1. pp. 23-26.
- Gupta, P. K., Dill, J. F., and Bandow, H. E. (1985): Dynamics of Rolling Element Bearings-Experimental Validation of the DREB and RAPIDREB Computer Programs, *J. Tribol.*, vol. 107, no. 1. pp. 132-137.
- Gupta, P. K. et al. (1986): Ball Bearing Response to Cage Unbalance. *J. Tribol.*, vol. 108, no. 3, pp. 462-467.
- Hamrock, B. J., and Anderson, W. J. (1983): Rolling-Element Bearings. *NASA Ref. Publ.-1105*.

- Hamrock, B. J., and Dowson, D. (1981): *Ball Bearing Lubrication-The Elastohydrodynamics of Elliptical Contacts*. Wiley-Interscience. New York.
- Harris, T. A. (1966): An Analytical Method to Predict Skidding in High Speed Roller Bearings. *ASLE Trans.*, vol. 9, pp. 229-241.
- Harris, T. A. (1971a): An Analytical Method to Predict Skidding in Thrust-Loaded, Angular Contact Ball Bearings. *J. Lubr. Technol.*, vol. 93, no. 1, pp. 17-21.
- Harris, T. A. (1971b): Ball Motion in Thrust-Loaded, Angular Contact Bearings With Coulomb Friction. *J. Lubr. Technol.*, vol. 93, no. 1. pp. 32-38.
- Heathcote, H. L. (1921): The Ball Bearing: In the Making, Under Test and in Service. *Proc. Inst. Auto. Eng. (London)*, vol. 15, pp. 569-702.
- Ioannides, E., and Harris, T. A. (1985): A New Fatigue Life Model for Rolling Bearings. *J. Tribol.*, vol. 107, no. 3, pp. 367-378.
- ISO (1976): Rolling Bearings, Dynamic Load Ratings and Rating Life. ISO / TC4 / JC8, Revision of ISOR281. Issued by International Organization for Standardization, Technical Committee ISO/TC4.
- Johnson, K. L., and Tabor, D. (1967-1968): Rolling Friction. Lubrication and Wear: Fundamentals and Application to Design. *Proc. Inst. Mech. Eng.*, vol. 182, pt. 3A, pp. 168-172.
- Jones, A. B. (1959): Ball Motion and Sliding Friction in Ball Bearings. *J. Basic Eng.*, vol. 81, no. 1. pp. 1-12.
- Jones, A. B. (1960): A General Theory for Elastically Constrained Ball and Radial Roller Bearings Under Arbitrary Load and Speed Conditions. *J. Basic Eng.*, vol. 82, no. 2. pp. 309-320.
- Jones, A. B. (1964): The Mathematical Theory of Rolling Element Bearings. *Mechanical Design and Systems Handbook*. H. A. Rothbart (ed.). McGraw-Hill, New York, pp. 13-1 to 13-76.
- Kunz, R. K., and Winer, W. O. (1977): Discussion on pp. 275-276 of Hamrock, B. J., and Dowson, D., Isothermal Elastohydrodynamic Lubrication of Point Contacts. Part III-Fully Flooded Results. *J. Lubr. Technol.*, vol. 99, no. 2, pp. 264-275.
- Lundberg, G., and Palmgren, A. (1947): Dynamic Capacity of Rolling Bearings. *Acta Polytech. Mech. Eng. Ser.*, vol. 1, no. 3. (Ingeniors Vetenskaps Akadamen Handlingar, no. 196.)
- Lundberg, G., and Palmgren, A. (1952): Dynamic Capacity of Roller Bearings. *Acta Polytech. Mech. Eng. Ser.*, vol. II, no. 4. (Ingeniors Vetenskaps Akadamen Handlingar, no. 210.)
- Matt, R. J., and Gianotti, R. J. (1966): Performance of High Speed Ball Bearings With Jet Oil Lubrication. *Lubr. Eng.*, vol. 22. no. 8, pp. 316-326.
- McCarthy, P. R. (1973): Greases. Interdisciplinary Approach to Liquid Lubricant Technology. P. M. Ku (ed.). *NASA Spec. Publ. 318*, pp. 137-185.
- Meeks, C. R., and Ng, K. O. (1985a): The Dynamics of Ball Separators in Ball Bearings-Part I: Analysis. *ASLE Trans.*, vol. 28, no. 3, pp. 277-287.
- Meeks, C. R., and Ng, K. O. (1985b): The Dynamics of Ball Separators in Ball Bearings-Part II: Results of Optimization Study. *ASLE Trans.*, vol. 28, no.

3, pp. 287-295.

- Miyakawa, Y., Seki, K., and Yokoyama, M. (1972): Study on the Performance of Ball Bearings at High DN Values. *NAL-TR-284*, National Aerospace Lab., Tokyo, Japan. (NASA TTF-15017, 1973).
- Palmgren, A. (1959): *Ball and Roller Bearing Engineering*, 3d ed. SKF Industries, Inc., Philadelphia.
- Parker, R. J. (1980): Lubrication of Rolling Element Bearings. in *Bearing Design-Historical Aspects, Present Technology and Future Problems*. W. J. Anderson (ed.). American Society of Mechanical Engineers, New York, pp. 87-110.
- Parker, R. J., and Signer, H. R. (1978): Lubrication of High-Speed, Large Bore Tapered-Roller Bearings. *J. Lubr. Technol.*, vol. 100, no. 1, pp. 31-38.
- Pirvics, J. (1980): Computerized Analysis and Design Methodology for Rolling Element Bearing Load Support Systems. *Bearing Design-Historical Aspects, Present Technology and Future Problems*. W. J. Anderson (ed.). American Society of Mechanical Engineers. New York, pp. 47-85.
- Reynolds, O. (1886): On Rolling Friction. *Philos. Trans. Roy. Soc. London*, part 1, vol. 166, pp. 155-174.
- Schuller, F. T. (1979): Operating Characteristics of a Large-Bore Roller Bearing to Speeds of 3×10^6 DN. *NASA Tech. Paper-1413*.
- Scibbe, H. W. (1968): Bearings and Seals for Cryogenic Fluids. *SAE Pap. 680550*. (NASA Tech. Memo. X-52415.)
- Signer, H., Bamberger, E. N., and Zaretsky, E. V. (1974): Parametric Study of the Lubrication of Thrust Loaded 120-mm Bore Ball Bearings to 3 Million DN. *J. Lubr. Technol.*, vol. 96, no. 3, pp. 515-524, 526.
- Signer, U. R., and Schuller, F. T. (1982): Lubrication of 35-Millimeter-Bore Ball Bearings of Several Designs at Speeds to 2.5 Million DN. Problems in Bearings and Lubrication, *AGARD Conf. Proc. 323*, pp. 8-1 to 8-15.
- Skurka, J. C. (1970): Elastohydrodynamic Lubrication of Roller Bearings. *J. Lubr. Technol.*, vol. 92, no. 2, pp. 281-291.
- Stribeck, R. (1901): Kugellager fur beliebige Belastungen. *Z. Ver. Dtsch. Ing.*, vol. 45, pp. 73-125.
- Tabor, D. (1955): The Mechanism of Rolling Friction. II. The Elastic Range. *Proc. Roy. Soc. London., Ser. A*, vol. 229, no. 1177, Apr. 21, pp. 198-220.
- Tallian, T. E. (1967): On Competing Failure Modes in Rolling Contact. *Trans. ASLE*, vol. 10, pp. 418-439.
- Tallian, T. E., Chiu, Y. P., and VanAmerongen, E. (1978): Prediction of Traction and Microgeometry Effects on Rolling Contact Fatigue Life. *J. Lubr. Technol.*, vol. 100, no. 2, pp. 156-166.
- Todd, M. J., and Bentall, R. H. (1978): Lead Film Lubrication in Vacuum. Proceedings of 2nd ASLE International Conference on Solid Lubrication, pp. 148-157.
- Walters, C. T. (1971): The Dynamics of Ball Bearings. *J. Lubr. Technol.*, vol. 93, no. 1, pp. 1-10.
- Weibull, W. (1949): A Statistical Representation of Fatigue Failures in Solids.

Trans. Roy. Inst. Technol., Stockholm, no. 27.

Zaretsky, E. V., Signer, H., and Bamberger, E. N. (1976): Operating Limitations of High-Speed Jet-Lubricated Ball Bearings. *J. Lubr. Technol.*, vol. 98, no. 1, pp. 32-39.